# Antisymmetric solitons and their interactions in strongly dispersion-managed fiber-optic systems

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## Abstract

By means of the variational approximation (VA), a system of ordinary differential equations (ODEs) is derived to describe the propagation of antisymmetric solitons in a multi-channel (WDM) optical fiber link subject to strong dispersion management. Results are reported for a prototypical model including two channels. Using the VA technique, conditions for stable propagation of the antisymmetric dispersion-managed (ASDM) solitons in one channel are found, and complete and incomplete collisions between the solitons belonging to the different channels are investigated. In particular, it is shown that formation of a bound inter-channel state of two ASDM solitons is possible under certain conditions (but may be easily avoided). The VA predictions for the single- and two-channel systems are compared with direct simulations of the underlying partial differential equations. In most cases, the agreement is very good, but in some cases (very closely spaced channels) the collision may destroy the ASDM solitons. The timing-jitter suppression factor (JSF) for the ASDM soliton in one channel, and the crosstalk timing jitter induced by collision between the solitons belonging to the different channels are also estimated analytically. In particular, the JSF for the ASDM soliton may be much larger than for its fundamental-soliton counterpart in the same system.

# 1 Introduction

The potential offered by the use of the dispersion management (DM), i.e., periodic compensation of the group-velocity dispersion in a long fiber-optic telecommunication link, for the improvement of data transmission by soliton streams, is well known [1]. The work in this direction, especially aimed at the application of DM to multi-channel systems based on the wavelength-division-multiplexing (WDM) technique, continues, see, e.g., a recent experimental report [2]. The interest to the topic has been recently bolstered by the development of the concept of the differential phase-shift keying (instead of the traditional on-off code), which helps to resolve some problems [3], and by the launch (in Australia) of the first commercial DM-soliton-based commercial fiber-optic telecommunications link.

Basic properties of the fundamental solitons in DM systems have been studied in detail by means of various analytical and numerical methods (some references are in given in more particular contexts below). It is known that, alongside the fundamental solitons which have a symmetric profile in the temporal domain, *antisymmetric* solitons are also possible in the DM fiber-optic links [4] [in contrast to the uniform nonlinear optical fibers, described by the nonlinear Schrödinger (NLS) equation, that does not give rise to antisymmetric solutions]. In the general case, the asymmetric DM (ASDM) solitons may be unstable against parity-breaking (symmetric) perturbations, but, nevertheless, in many respects they behave as fairly robust pulses, that is why they are of interest to applications [5]. Besides that, they are interesting dynamical objects in their own right – in particular, because they are related to the so-called "twisted localized modes" (TLM; alias "dark-in-bright solitons") in Bose-Einstein condensates (BECs) loaded in a periodic potential ( optical lattice) [6], and recently found odd solitons in BECs subjected to the Feshbach-resonance management (time-periodic change of the sign of the nonlinearity constant, under the action of external ac magnetic field) [7]. Those solitons, in turn, were found following the pattern of earlier found TLMs in the discrete NLS equation [8]. It is relevant to mention that the TLM pulses in Bose-Einstein condensates are completely stable objects, including full stability against parity-violating perturbations [6].

The study of the ASDM solitons in single- and two-channel systems is the subject of this work. First, we aim to develop the variational approximation (VA) for the antisymmetric solitons in the single-channel DM model, in order to predict conditions for stable transmission of these solitons in the long DM fiber link. Then, we extend the VA for the case of interactions (collision) between the ASDM solitons in the two-channel system. These analytical results are presented in Section 2, and in Section 3 they are verified versus direct numerical simulations. We infer that, in most cases, the VA predictions are quite accurate, except for a case of two very close channels, when collision may completely destroy the solitons. In Section 4, we produce analytical estimates for the intra-channel jitter-suppression factor (JSF),

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and for the crosstalk jitter induced by complete and incomplete collisions between the ASDM solitons belonging to different channels. A noteworthy result is that, for the antisymmetric solitons, the JSF may be much larger (by a factor in excess of 10) than for their fundamental counterparts. The paper is concluded by Section 5.

# 2   The analytical approach

## 2.1   The models

We take the propagation equation for the DM transmission line in the following standard normalized form (see, e.g., Refs. [9, 10]):

$$2iu_z + D(z)u_{\tau\tau} + \epsilon(D_0 u_{\tau\tau} + 2|u|^2 u) = 0, \tag{1}$$

where $u(z,\tau)$ is the envelope of the electromagnetic field, $z$ and $\tau$ are the propagation distance along the fiber and the retarded time, respectively, and $D(z)$ is the local dispersion coefficient, which is a periodic function with a period $L_{\mathrm{map}} \equiv L_1 + L_2$:

$$D(z) = \begin{cases} D_1, & 0 < \mathrm{mod}(z, L_{\mathrm{map}}) < L_1, \\ D_2, & L_1 < \mathrm{mod}(z, L_{\mathrm{map}}) < L_1 + L_2. \end{cases} \tag{2}$$

The map (2) is subject to the *dispersion-compensation condition*, $D_1 L_1 + D_2 L_2 = 0$, and its parameters may be rescaled to satisfy the following normalizations [11],

$$L_1 + L_2 = 1, \; |D_1|L_1 = |D_2|L_2 = 1. \tag{3}$$

The small parameter $\epsilon$ in Eq. (1) is the ratio of the local dispersion length to the nonlinear length, which measures the weakness of the nonlinearity. The coefficient $D_0$ is the path-average dispersion (PAD); its positive, zero, or negative values correspond, respectively, to the anomalous, zero and the normal average dispersion, respectively. The form of Eq. (1) implies that the nonlinearity and PAD are weak factors at the same order of smallness.

A two-channel system for the fields $u(z,\tau)$ and $v(z,\tau)$ propagating in the same core obeys coupled nonlinear Schrödinger equations,

$$2i\left(u_z + \overline{c}u_\tau\right) + D(z)u_{\tau\tau} + \epsilon\left[\overline{D}_u u_{\tau\tau} + 2\left(|u|^2 + 2|v|^2\right)u\right] = 0, \tag{4}$$

$$2i\left(v_z - \overline{c}v_\tau\right) + D(z)v_{\tau\tau} + \epsilon\left[\overline{D}_v v_{\tau\tau} + 2\left(|v|^2 + 2|u|^2\right)v\right] = 0, \tag{5}$$

where $2\overline{c}$ is the inverse group velocity difference between the channels, $D(z)$ is the same periodic map as in Eq. (2), while the two PAD coefficients $\overline{D}_{u,v}$ may be different. Nonlinear terms in Eqs. (1) and (2) represent the self-phase modulation

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(SPM) and cross-phase modulation (XPM), which are induced by the Kerr effect [12]. Equations (4), (5) conserve two channel energies,

$$E_u \equiv \sqrt{2/\pi} \int_{-\infty}^{+\infty} |u|^2 d\tau, \; E_v \equiv \sqrt{2/\pi} \int_{-\infty}^{+\infty} |v|^2 d\tau \qquad (6)$$

(the factor $\sqrt{2/\pi}$ was added to the definitions for convenience, see below), as well the Hamiltonian and field momentum.

It is straightforward to establish relations between the normalized variables and parameters used in the above equations, and their physical counterparts. In particular, if $z = 1$ and $\tau = 1$ correspond to the typical values, 50 km and 10 ps, respectively, which frequently play the role of the length ant time units in fiber-optic telecommunications, then $D = 1$ corresponds to the actual value of the dispersion coefficient 2 ps$^2$/km (in uniform links, this value is typical for dispersion-shifted fibers), and $\overline{c} = 1$ corresponds to 0.2 ps/km. With the above-mentioned value of the dispersion coefficient, 2 ps$^2$/km, the inverse-group-velocity difference of 0.2 ps/km between the channels implies the wavelength separation $\Delta\lambda \approx 0.15$ nm between them, which corresponds to the case of dense WDM.

The single-channel model (1) was used for the derivation of conditions for stable transmission of fundamental DM solitons [11, 13], and for the study of higher-order DM pulses based on the Hermite-Gaussian functions [14]. The two-channel model (4), (5) was used to investigate inter-channel collisions between fundamental pulses [15]. Under certain conditions, the two-channel model can also predict formation of bound states between two fundamental solitons belonging to different channels, which was investigated in Ref. [16].

## 2.2 The variational approximation for antisymmetric solitons

The Hermite-Gaussian set of functions can be used to describe the propagation of pulses of a general shape in the strong-DM regime, the fundamental soliton of the Gaussian form being the first term in the set [14]. The antisymmetric DM (ASDM) soliton corresponds to the second function belonging to the set, so that it can be approximated by the following variational ansatz:

$$u(z, \tau) = A\tau \exp\left(-\frac{\tau^2}{W^2} + ib\tau^2 + i\phi\right) . \qquad (7)$$

Here, $A$, $W$, $b$ and $\phi$ represent the amplitude, width, chirp and phase of the pulse, respectively, and they are allowed to be functions of $z$. The pulse is called the antisymmetric soliton because $|u(z, \tau)|$ is an odd function of $\tau$. The energy of the pulse (7), calculated according to the definition (6), is $E = A^2 W^3/4$.

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The antisymmetric soliton may also be represented in an alternative form,

$$u(z, \tau) = A\tau \exp\left(-\frac{\tau^2}{\tau_0{}^2 + 2i\Delta} + i\phi\right),\tag{8}$$

where $\Delta(z) \equiv \int_0^z D(z')\,dz' + \Delta_0$ is the accumulated dispersion, and $\tau_0$ is the minimum width of the pulse over the DM period. They are related to the parameters of $W$ and $b$ from the ansatz (7),

$$W = \frac{\sqrt{\tau_0{}^4 + 4\Delta^2}}{\tau_0}, \qquad b = \frac{2\Delta}{\tau_0{}^4 + 4\Delta^2}.\tag{9}$$

Following the commonly adopted definition [9], below we will be using, instead of $\tau_0$, the so-called DM strength,

$$S \equiv 1.443/\tau_0^2.$$

The Lagrangian of the system (4), (5) is $L = \int_{-\infty}^{+\infty} \mathcal{L}\,dt$, with the Lagrangian density

$$
\begin{aligned}
\mathcal{L} &= \frac{i}{2}\left[(u_z u^* - u u_z^* + v_z v^* - v v_z^*) + \bar{c}\left(u_\tau u^* - u u_\tau^* - v_\tau v^* + v v_\tau^*\right)\right]\\
&\quad -\frac{1}{2}D(z)\left(|u_\tau|^2 + |v_\tau|^2\right) - \frac{\epsilon}{2}\left(\overline{D}_u|u_\tau|^2 + \overline{D}_v|v_\tau|^2\right)\\
&\quad +\frac{\epsilon}{2}\left(|u|^4 + |v|^4 + 4|u|^2|v|^2\right).
\end{aligned}\tag{10}
$$

Applying the known technique of the VA for pulses of the Gaussian type [10], and skipping routine technical details, we obtain the following system of ordinary differential equations (ODEs) which govern the evolution of parameters of the ansatz (7) in the single-channel system (with $\bar{c} = 0$):

$$\frac{dE}{dz} = 0,\tag{11}$$

$$\frac{dW}{dz} = 2[D(z) + \epsilon D_0]\,bW,\tag{12}$$

$$\frac{db}{dz} = 2[D(z) + \epsilon D_0]\left(\frac{1}{W^4} - b^2\right) - \frac{\sqrt{2}}{8W^3}\epsilon E.\tag{13}$$

where Eq. (11) simply means that the energy is conserved. From Eqs. (9), the evolution ODEs for $\Delta_0$ and $\tau_0$ can also be derived:

$$\frac{d\tau_0}{dz} = \frac{\sqrt{2}}{16}\frac{\epsilon E \tau_0 \Delta}{W^3},\tag{14}$$

$$\frac{d\Delta_0}{dz} = \epsilon D_0 + \frac{\sqrt{2}}{16}\frac{\epsilon E(4\Delta^2 - \tau_0^4)}{W^3}.\tag{15}$$

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Conditions for stationary propagation of the ASDM soliton can be obtained in the same way as it was done for the fundamental DM soliton in Ref. [11]. To this end, one should demand that the pulse's amplitude and width return to the original values after passing one DM period, i.e., $\tau_0(z) = \tau_0(z+1)$ and $\Delta_0(z) = \Delta_0(z+1)$. In the first-order approximation (which implies that the parameters $\tau_0$ and $\Delta_0$ suffer a small variation within one period), these conditions amount to

$$\int_0^1 \frac{d\tau_0}{dz} dz = \int_0^1 \frac{d\Delta_0}{dz} dz = 0. \tag{16}$$

Substituting Eqs. (14), (15) into Eqs. (16), and evaluating some integrals explicitly, we obtain

$$\Delta_0 = -1/2, \tag{17}$$

$$D_0 = -\frac{\sqrt{2}}{16} E\tau_0{}^3 \left[ \ln\left(\sqrt{1 + \tau_0{}^{-4}} + \tau_0{}^{-2}\right) - 2\left(\tau_0{}^4 + 1\right)^{-1/2} \right]. \tag{18}$$

Note that the simple result (17) is exactly the same as for the fundamental solitons [11], which implies that the pulse has zero chirp at the midpoint of each fiber segment. Condition (18) is also similar to the corresponding condition for the fundamental solitons, as derived in Ref. [11], only differing by a factor of $1/4$.

Straightforward extension of the VA-based analysis performed in Ref. [11] for the fundamental DM solitons, we arrive at the following conclusions for their antisymmetric counterparts:

1. stable ASDM solitons exist at zero PAD if $S \approx 4.79$.

2. stable ASDM solitons exist at anomalous PAD if $S < 4.79$.

3. stable ASDM solitons exist at normal PAD if $4.79 < S < 9.75$ and $|\overline{D}|/E \leq 0.0032$.

Before proceeding further, we make several remarks. First, the stable antisymmetric soliton is possible at both the anomalous PAD and normal PAD, as well as when the PAD is zero. Second, the detailed VA analysis shows that the energy of the stable antisymmetric soliton is four times as large as that for the fundamental DM soliton with the same width; as is well known, the "heavier" soliton provides for better suppression of the timing jitter, so the antisymmetric one has advantage, in this respect for applications to fiber-optic telecommunications (see further details in section 4). Third, the VA predictions are completely verified by direct simulations, see below.

## 2.3   Interactions between antisymmetric solitons

To describe the interaction between pulses belonging to different channels, we start with a more general expression for the pulse, which is obtained from the one-channel solution by the Galilean boost,

$$
\begin{aligned}
u(z,\tau) &= u_0\left(z, \tau - T_u(z)\right)\exp\left(-i\omega_u(\tau - T_u) + i\psi_u(z)\right), \\
v(z,\tau) &= v_0\left(z, \tau - T_v(z)\right)\exp\left(-i\omega_v(\tau - T_v) + i\psi_v(z)\right).
\end{aligned}
\tag{19}
$$

Here $T_{u,v}$, $\psi_{u,v}$ and $\omega_{u,v}$ are, respectively, the position, phase, and frequency shifts. In terms of the Galilean boost, the latter are constant, while the position shifts evolve in $z$ according to the equations

$$
\frac{dT_{u,v}}{dz} = \pm c - \left(D(z) + \epsilon\overline{D}_{u,v}\right)\omega_{u,v},
\tag{20}
$$

which also include a contribution from the group-velocity difference between the channels.

The application of the VA technique to the two pulses defined as in Eq. (7) leads to the following results: the two energies $E_{u,v} \equiv (1/4)A_{u,v}^2 W_{u,v}^3$ are conserved separately in the channels, and the other variational parameters evolve according to the following ODEs

$$
\frac{dW_{u,v}}{dz} = 2\left[D(z) + \epsilon\overline{D}_{u,v}\right]W_{u,v}b_{u,v};
\tag{21}
$$

$$
\frac{d\omega_{u,v}}{dz} = \frac{\pm 32\epsilon E_{v,u}(T_u - T_v)}{(W_u^2 + W_v^2)^{5/2}}\exp\left[-\frac{2(T_u - T_v)^2}{W_u^2 + W_v^2}\right]B,
\tag{22}
$$

here we define

$$
\begin{aligned}
B &= \frac{2\left(W_u^2 - W_v^2\right)^2 - 7W_u^2 W_v^2}{4\left(W_u^2 + W_v^2\right)} - \frac{\left(W_u^2 - W_v^2\right)^2 - 6W_u^2 W_v^2}{\left(W_u^2 + W_v^2\right)^2}(T_u - T_v)^2 \\
&\quad - \frac{4W_u^2 W_v^2}{\left(W_u^2 + W_v^2\right)^3}(T_u - T_v)^4.
\end{aligned}
$$

Note that, as it follows from Eq. (22),

$$
\frac{d}{dz}\left[E_u\omega_u + E_v\omega_v\right] = 0,
\tag{23}
$$

which implies the conservation of the net momentum, $P \equiv E_u\omega_u + E_v\omega_v$.

Now, we focus on the most interesting case when the pulse in each channel is a stable antisymmetric soliton of the same width, i.e., in the absence of the interaction between them, their parameters are selected according to Eqs. (17) and (18). Then

the interaction between the two antisymmetric solitons is described by a system including a difference of two equations Eq. (20),

$$\frac{d}{dz}\left(T_u - T_v\right) = 2\overline{c} - D(z)\left(\omega_u - \omega_v\right) - \epsilon\left(\overline{D}_u\omega_u - \overline{D}_v\omega_v\right). \tag{24}$$

and Eqs. (22) that can be now cast in the form

$$\frac{d\omega_{u,v}}{dz} = \frac{\pm 4\sqrt{2}\epsilon E_{v,u}\tau_0^3\left(T_u - T_v\right)}{\left[\tau_0^4 + 4\Delta^2(z)\right]^{3/2}}\exp\left(-\frac{\left(T_u - T_v\right)^2\tau_0^2}{\tau_0^4 + 4\Delta^2(z)}\right)C, \tag{25}$$

$$C \equiv -\frac{7}{8} + \frac{3\tau_0^2\left(T_u - T_v\right)^2}{2\left[\tau_0^4 + 4\Delta^2(z)\right]} - \frac{\tau_0^4\left(T_u - T_v\right)^4}{2\left[\tau_0^4 + 4\Delta^2(z)\right]^2}$$

Equations (25) and (24) constitute a dynamical system describing the interaction between the antisymmetric solitons and formation of possible bound states between them, similar to how it was investigated for fundamental DM solitons in Ref. [16]. Note that the energies $E_{u,v}$ do not appear in these equations as arbitrary parameters; instead, they must be expressed in terms of $\tau_0$ and $\overline{D}_{u,v}$ by means of Eqs. (17) and (18). Arbitrary parameters are $\tau_0$, or the DM strength $S$, the inverse-group-velocity-difference $\overline{c}$, and the PADs $\overline{D}_{u,v}$.

The third-order system of Eqs. (25) and (24) can be further reduced to a second-order one in the symmetric case, with $\overline{D}_u = \overline{D}_v$ [hence also $E_u = E_v$, see Eqs. (17),(18)]. Then, defining $T \equiv T_u - T_v$, $\omega \equiv \omega_u - \omega_v$, and $E_u = E_v \equiv E$, $\overline{D}_u = \overline{D}_v \equiv D$, the reduced system is

$$\frac{d\omega}{dz} = \frac{8\sqrt{2}\epsilon E\tau_0^3 T}{\left[\tau_0^4 + 4\Delta^2(z)\right]^{3/2}}\exp\left(-\frac{\tau_0^2 T^2}{\tau_0^4 + 4\Delta^2(z)}\right)C, \tag{26}$$

$$\frac{dT}{dz} = 2\overline{c} - \left[D(z) + \epsilon\overline{D}\right]\omega. \tag{27}$$

It should be pointed out here that, if PAD is zero, it may be necessary to add the third-order dispersion (TOD) to the DM model, in the case when the solitons are taken very narrow (in the temporal domain), to provide for a very high bit rate per channel. Effects of TOD on fundamental DM solitons have been systematically studied in [17]. It was shown in that the TOD gives rise to an asymmetry of the DM-soliton's profile and generation of radiation. We anticipate that the effects of TOD on antisymmetric solitons will be similar. However, detailed investigation on this issue is definitely beyond the scope of the present paper, being a subject for a separate work.

# 3 Comparison with the results of direct simulations

It is necessary to check the VA equations derived in previous section against direct simulations. To this end, we solved the underlying equations (1) and (4), (5) by a symmetrized split-step Fourier method, in which the linear part is computed exactly via the fast Fourier transformation (FFT), and the nonlinear part is evaluated implicitly via an iteration procedure at the midpoint of the stepsize (see, e.g., Ref. [16]).

## 3.1 Single ASDM soliton

First of all, the validity of Eqs. (17) and (18), which predict equilibrium values of the parameters for the ASDM soliton, Eq. (1) was solved numerically with the parameters of the initial antisymmetric pulse taken as predicted by these expressions. We fixed $L_1 = L_2 = 0.5$, and $D_1 = 2.0$, $D_2 = -2.0$, unless specified otherwise.

In the zero-PAD case, direct simulations show that the width parameter of the pulse is always kept close to $\tau_0^2 = 0.301$ regardless of the value of energy $E$. An example is shown in Fig. 1 for $E = 1.0$ and $E = 2.0$. It can be seen that the ASDM soliton remains stable, keeping the same width as the initial pulse at the end of each dispersion segment.

In the anomalous-PAD case, results of direct simulation agree with the VA predictions as well. Figure 2 shows the evolution of the antisymmetric solitons for $\overline{D} = 0.05$, and $E = 2.0, 4.0$. The corresponding widths are $\tau_0^2 = 0.6993$ and $0.5256$, respectively.

In the normal-PAD case, the VA predicts that the antisymmetric DM soliton is stable only when $|\overline{D}|/E \leq 0.0032$. To test this, we took, for instance, $\overline{D} = -0.01$ and $E = 2.0, 4.0$. The soliton is anticipated to be unstable for $E = 2.0$, since $|\overline{D}|/E = 0.005$ in this case, and stable for $E = 4.0$, as then $|\overline{D}|/E = 0.0025$. These predictions are confirmed by direct simulations, whose results are shown in Fig. 3 (solid and dashed curves showing, respectively, the wave profile at $z = 800$ and the initial one). It is seen that, for $E = 2.0$, the two parts of the soliton separate from each other as $z$ increase, but for $E = 4.0$ the amplitudes of the two peaks and the difference between them keep almost the same values as they had in the initial pulse, even as $z$ takes attains the large value of 800, which is only violated by some radiation loss.

To summarize these results, in Fig. 4 we plot the ratio of the PAD to the pulse energy, $\overline{D}/E$, versus the pulse's width $\tau_0^2$ for the cases of the zero, anomalous, and normal PAD. The solid curves depict the VA predictions, while the circles are data produced by direct simulations. Good agreement between the VA and numerical results is obvious.

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## 3.2 Interactions between antisymmetric solitons and formation of bound states

Proceeding to interactions between the antisymmetric solitons, we first simulated head-on complete collisions, in which case the pulses, moving with opposite velocities, are well separated before and after the collision. Basically, the collision features the generic property of the soliton collision, that is, the pulses pass through each other with position shifts. However, because the DM pulses considered here are not solitons in the strict mathematical sense, each one gets slightly distorted by the interaction, its humps changing their height. A typical example is displayed in Fig. 5 for $\overline{D}_u = \overline{D}_v = 0.05$, $2\overline{c} = 0.1$, and $E_u = E_v = 2.0$. In the direct simulation, it is observed that the two pulses repel each other at an early stage of the interaction, and attract at a late stage. A position shift of $\delta T = 0.625$ has resulted from the complete collision. This result agree with the VA results shown in Fig. 5(c), which predict the position shift 0.65 and the zero frequency shift for each pulse. The position shift, along with a possible frequency shift may be considered as the source of the timing jitter induced by the collisions, which will be considered in more detail in the next section.

It is predicted by VA that there is no possibility for the formation of BS's in the case of complete collision. As to the case of incomplete collision, similar to the possibility of formation of bound states (BS's) of two fundamental DM soliton belonging to different channels that was found in Ref. [16], BS's of the ASDM solitons can be formed too. The difference is that more energy is needed for the formation of BSs in the latter case. Figure 6 shows an example for $\overline{D}_u = \overline{D}_v = 0.075$, $2\overline{c} = 0.05$, $E_u = E_v = 3.0$, two antisymmetric solitons being initially set at the same position. Since the formation of BSs is detrimental for the fiber-optic telecommunication systems, the smaller chance for this effect in the case of the antisymmetric solitons is an advantage offered by them.

For incomplete collisions in the symmetric situation, with $D_u = D_v = D$, $E_u = E_v = E$, and $D/E = 0.025$, a plot of the minimum energy $E_{\min}$, necessary for the formation of the BS, vs. $2\overline{c}$, as predicted by VA is displayed in Fig. 7. The variational predictions are checked, at several points, against direct simulations, the corresponding data being marked by rhombuses. As is seen, the agreement between VA and direct results is good. It is noted that both the VA predictions and direct simulations yield a critical value of $2\overline{c}_r \approx 0.2$, above which no BSs exist, no matter how large the energy is. In other words, the formation of a BS is prevented for the values of IGVD exceeding $2\overline{c}_r$.

However, in some cases the ASDM solitons may be completely distorted by the interaction, see an example in Fig. 8 for $\overline{D}_u = \overline{D}_v = 0$, $2\overline{c} = 0.1$, and $E_u = E_v = 4.0$. This phenomenon often happens when the energy is large or the group-velocity difference between the channels is small, which are detrimental features for the applications.

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We also simulated collisions between fundamental and antisymmetric solitons, see a typical example in Fig. 9. It is seen that both the fundamental and antisymmetric solitons do not change their shapes after the collision. A theoretical study of interactions between the fundamental and antisymmetric DM solitons could be a natural extension of the present work.

# 4 Timing jitter of antisymmetric solitons

## 4.1 Estimate of the timing-jitter suppression in one channel

Based on the variational results for the antisymmetric solitons presented above, we now aim to estimate the Gordon-Haus timing jitter (generated by optical noise in the fiber link [12]) for pulses of this type. We will follow the procedure of evaluating the jitter which was implemented for fundamental DM soliton in Ref. [11]. To this end, we use a known expression for the jitter-suppression factor (JSF) for the DM soliton vis-a-vis its NLS counterpart, provided the two have equal energies (see details in Refs. [11] and [16]):

$$\text{JSF} = \frac{\left(\int_{-\infty}^{\infty} \tau^2 |u_0|^2\right)_{\text{DM}}}{\left(\int_{-\infty}^{\infty} \tau^2 |u_0|^2\right)_{\text{NLS}}} \ . \tag{28}$$

Using the analytical approximation (8) for the antisymmetric soliton, we find

$$\text{JSF} = -\frac{36}{\pi^{3/2}} \frac{\tau_0^4 + 1/3 + (\Delta_0 + 1)^2}{\tau_0^6 \left[\ln\left(\sqrt{1 + \tau_0^{-4}} + \tau_0^{-2}\right) - 2\left(\tau_0^4 + 1\right)^{-1/2}\right]} \ . \tag{29}$$

For comparison, JSF for the fundamental DM soliton is [11]

$$\text{JSF} = -\frac{3}{\pi^{3/2}} \frac{\tau_0{}^4 + 1/3 + (\Delta_0 + 1)^2}{\tau_0{}^6 \left[\ln\left(\sqrt{1 + \tau_0^{-4}} + \tau_0^{-2}\right) - 2\left(\tau_0^4 + 1\right)^{-1/2}\right]}.$$

Another characteristic of the DM solitons is the stretching factor ( SF), which is the ratio the maximum and minimum values of its temporal width,

$$\text{SF} \equiv \sqrt{\tau_0^4 + (1 + (2\Delta_0 + 1))^2}/\tau_0^2 \ . \tag{30}$$

A certain compromise between the JSF and SF must be reached in designing a transmission line for DM solitons. In Fig. 10, JSF is plotted versus SF for both the fundamental (dashed curve) and antisymmetric (solid curve) DM solitons. It can be seen that much higher (four times) energy is needed to support the same width of the antisymmetric soliton, in comparison with the fundamental one. As a result, the JSF for the antisymmetric soliton is 12 times larger than for the fundamental one. This is a potential advantage for using ASDM solitons in fiber-optic telecommunications.

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## 4.2 Collision-induced pulse timing jitter

One of the serious problems in the use of multi-channel (WDM) schemes is the crosstalk timing jitter, induced by collisions of pulses belong to different channels. Here, we aim to estimate the crosstalk jitter induced by collisions between antisymmetric DM solitons belonging to two adjacent channels, in the case of both complete and incomplete collisions.

We will follow the approach to this problem developed for the fundamental DM pulses in Ref. [15]. Straightforward use of general expressions for the collision-induced frequency and position shifts, $\delta\omega_{u,v}$ and $\delta T_{u,v}$ produced by the collision, which were derived in that work, yields the following results for the ASDM solitons. In the lowest approximation, $\delta\omega_{u,v} = 0$, and

$$\delta T_u = \frac{\sqrt{2\pi}\epsilon^2 \overline{D}_u E_v}{2\,\overline{c}^2} \tag{31}$$

For a typical example corresponding to Fig. 5 (see above), with $\overline{D}_u = \overline{D}_v = 0.05$, $2\overline{c} = 0.1$, $E_u = E_v = 2.0$, Eq. (31) yields $\delta T_u = 0.501$. This result agrees well with that produced by numerical integration of the full VA equations (26) and (27), as well as with direct simulations of the underlying equations (4) and (5), as is seen in Fig. 5.

For incomplete collisions, in which two pulses are initially overlapped, the general formulas borrowed from Ref. [15] yield the following result for the largest size of the frequency shift, corresponding to the worst case, when the solitons begin their interaction at the point where their centers coincide:

$$(\delta\omega)_u^{\max} = -2\sqrt{2}\epsilon E_v \tau_0^3 \overline{c}^{-1} \left\langle \left(\tau_0^4 + 4\Delta^2\right)^{-3/2} \right\rangle, \tag{32}$$

with $\langle...\rangle$ standing for the average over the DM period. After evaluating the average value, we obtain from Eq. (32),

$$(\delta\omega)_u^{\max} = -2\sqrt{2}\epsilon E_v \tau_0^{-1} \overline{c}^{-1} \left( \frac{\Delta_0 + 1}{\sqrt{\tau_0^4 + 4(\Delta_0 + 1)^2}} - \frac{\Delta_0}{\sqrt{\tau_0^4 + 4\Delta_0^2}} \right). \tag{33}$$

Assuming that parameters for the antisymmetric solitons are selected as for the stationary pulses in one channel, i.e., $\Delta_0 = -1/2$, Eq. (33) is simplified to

$$(\delta\omega)_u^{\max} = -\frac{2\sqrt{2}\epsilon E_v}{\overline{c}\tau_0\sqrt{\tau_0 + 1}} . \tag{34}$$

Then, for a large propagation distance $z$, the position shift generated by the frequency shift grows as $\delta T_u^{(\omega)} = -\delta\omega\epsilon D_u z$.

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# 5    Conclusion

In this paper, we have studied the propagation and interactions of antisymmetric solitons in a fiber-optic link subject to strong DM. By means of the variational approximation (VA), we have obtained analytical expressions for the initial chirp and width of the antisymmetric pulse at which the pulse should propagate stably. Interactions between ASDM solitons belonging to two adjacent channels were also investigated, including the possibility of the formation of bound states between them. In most cases, the results predicted by the VA compare quite well with direct simulations for the underlying partial differential equations. However, in some cases we the collision between the ASDM solitons may destroy them, which is of course not predicted by the VA.

We have also estimated the Gordon-Haus timing jitter for the ASDM solitons. A noteworthy finding is that the jitter-suppression factor for the antisymmetric solitons may be much larger (by a factor of 12) than its previously known counterpart for the fundamental solitons in the same DM link. The crosstalk jitter, induced by inter-channel collisions between the antisymmetric solitons in a WDM system, was evaluated too. For complete collisions, the frequency shift is negligible, whereas the position shift is significant. Incomplete collisions are most dangerous, generating a finite frequency shift, which was estimated.

Results reported in this work suggest further investigation in several directions. In particular, as it was briefly mentioned above, it would be relevant to study how higher-order effects, such as the TOD and the intrapulse stimulated Raman scattering, act on the antisymmetric DM solitons. Interactions between fundamental and antisymmetric DM solitons, as well as a possibility of formation of bound states between them, may be another issue to be considered in the future. Lastly, for practical applications to WDM schemes, it would be useful to study multi-channel systems, rather than only the dual-channel one.

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# Figure Captions

Fig. 1. The profiles of the stable ASDM soliton (shown is $|u(\tau)|$), as found from direct simulations of Eq. (1) in the case of the zero path-average dispersion ($\overline{D} = 0$), for (a) $E = 1.0$; (b) $E = 2.0$. Dashed curve: $z = 0$; solid curve: $z = 800$.

Fig. 2. The same as in Fig. 1 (except for that the solid curve pertains to $z = 400$) in the case of anomalous path-average dispersion, with $D_0 = 0.05$, and (a) $E = 2.0$; (b) $E = 4.0$.

Fig. 3. The same as in Fig. 1 in the case of normal path-average dispersion with $D_0 = -0.01$, and (a) $E = 2.0$ and (b) $E = 4.0$, which correspond to the stable and unstable antisymmetric solitons, respectively.

Fig. 4. The ratio of the PAD to the pulse's energy versus its width in the cases of the zero, anomalous, and normal path-average dispersion. Solid curve: VA prediction; circles: numerical results.

Fig. 5. A typical example of the complete collision between two ASDM solitons with $E_u = E_v = 2.0$, $\overline{D}_u = \overline{D}_v = 0.05$, and $2\overline{c} = 0.1$. (a) The shape of $|u|$ at $z = 0$ (dashed curve) and $z = 400$ (solid curve); (b) the shape of $|v|$ at $z = 0$ (dashed curve) and $z = 400$ (solid curve); (c) the evolution of $\omega_u - \omega_v$ (dashed curve) and $T_u - T_v$ (solid curve) as predicted by the VA.

Fig. 6. A bound state of ASDM solitons with $\overline{D}_u = \overline{D}_v = 0.075$, $2\overline{c} = 0.1$ and $E_u = E_v = 3.0$, found from direct simulations of Eqs. (4) and (5). The panels (a) and (b) show the shapes of the bound solitons at $z = 400$ (solid curve) and the initial profile (dashed curve) for $|u(\tau)|$ and $|v(\tau)|$, respectively.

Fig. 7. The minimum energy necessary for the formation of a bound state of two antisymmetric solitons in the two-channel system vs. the inverse-group-velocity difference $2\overline{c}$ in the case of incomplete collisions in a symmetric situation, with $D_u = D_v = D$, $E_u = E_v = E$, and $D/E = 0.025$. The minimum energy predicted by VA is shown by the solid line. Rhombuses represent data points collected from direct simulations.

Fig. 8. An example of destruction of the ASDM solitons as a result of the collision with $\overline{D}_u = \overline{D}_v = 0.0$, $2\overline{c} = 0.1$ and $E_u = E_v = 4.0$, as found from direct simulations of Eqs. (4) and (5). The panels (a) and (b) show the shapes of $|u(\tau)|$ and $|v(t)|$ at $z = 0$ and $z = 400$.

Fig. 9. An example of the collision between fundamental and ASDM solitons with $E_u = E_v = 2.0$, $\overline{D}_u = \overline{D}_v = 0.05$ and $2\overline{c} = 0.2$. The panels (a) and (b) show the shapes of $|u|$ and $|v|$ for the fundamental and asymmetric solitons, respectively, at $z = 0$ (dashed curve) and $z = 200$ (solid curve).

Fig. 10. The jitter suppression factor versus the stretching factor for the fundamental (dashed) and antisymmetric (solid) DM solitons.

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