

## QUATERNION DYNAMICS OF THE BRAIN

E. A. Novikov

Institute for Nonlinear Science, University of California - San Diego, La Jolla, CA 92093 - 0402

A nonlinear dynamical modeling of interaction between automatic and conscious processes in the brain is described. Effects of sensations, emotions and reflections on the electromagnetic activity of the brain are represented in terms of quaternion field.

In previous paper [1] an approach to nonlinear dynamical modeling of interaction between automatic (A) and conscious (C) processes in the brain was presented. The idea is to use complex field with real and imaginary components representing A- and C-processes. The interaction is due to the nonlinearity of the system. This approach was illustrated on the nonlinear equation for the current density in the cortex. The nonlinearity is determined by the sigmoidal firing rate of neurons. More general approaches were also indicated [1]. In this letter the quaternion (Q) approach is described.

Subjective C-experiences can be divided into three major groups: sensations (S), emotions (E) and reflections (R). Note, that subjective S should be distinguished from the automatic sensory input into the neuron system of the brain. The effects of S, E and R on the electrochemical activity of the brain, generally, can be different. These effects should be distinguished in designing experiments and in C-modeling. The natural generalization of the approach [1] is to use the Q-field for representation of S-, E- and R-effects.

Let us illustrate this on the model equation for the average (spatially uniform) current density  $\alpha(t)$  perpendicular to the cortical surface [1]:

$$\frac{\partial \alpha}{\partial t} + k\alpha = f(\alpha + \sigma) \quad ((1))$$

Here  $k$  is the relaxation coefficient,  $\sigma(t)$  is the average sensory input and  $f$  represents the sigmoidal firing rate of neurons, for example,  $f(\alpha) = \tanh(\alpha)$ . For the case of spatially nonuniform  $\alpha(t, \mathbf{x})$  and  $\sigma(t, \mathbf{x})$  we can use a more general equation, which include typical propagation velocity of signals  $v$  [1]:

$$\frac{\partial^2 \alpha}{\partial t^2} + (k + m)\frac{\partial \alpha}{\partial t} + (km - v^2 \Delta)\alpha = (m + \frac{\partial}{\partial t})f(\alpha + \sigma) \quad ((2))$$

Here  $m$  is an additional parameter and  $\Delta$  is the two-dimensional spatial Laplacian. This type of equations are used for interpretation of the EEG and MEG spatial patterns (see recent paper [2] and references therein). In this context we have parameters:  $k \sim m \sim v/l$ , where  $l$  is the connectivity scale.

Let us now introduce the Q-field:

$$q = \alpha + i_p \psi_p \quad ((3))$$

Here components  $\psi_p$  represent (S, E, R)-effects and summation is assumed on repeated subscripts from 1 to 3. The quaternion imaginary units  $i_p$  satisfy conditions:

$$i_p i_q = \varepsilon_{pqr} i_r - \delta_{pq} \quad ((4))$$

where  $\varepsilon_{pqr}$  is the unit antisymmetric tensor and  $\delta_{pq}$  is the unit tensor. Formula (4) is a compact form of conditions:  $i_1^2 = i_2^2 = i_3^2 = -1$ ,  $i_1 i_2 = -i_2 i_1 = i_3$ ,  $i_2 i_3 = -i_3 i_2 = i_1$ ,  $i_3 i_1 = -i_1 i_3 = i_2$ . Conjugate Q-field is defined by:  $\bar{q} = \alpha - i_p \psi_p$ . From (3) and (4) it follow that the components and modulus of Q-field can be obtained by formulas:

$$\alpha = \text{Re}\{q\} = \frac{1}{2}(q + \bar{q}), \quad \psi_p = \text{Im}_p\{q\} = -\text{Re}\{q i_p\}, \quad \alpha^2 + \psi_p^2 = |q|^2 = q\bar{q} \quad ((5))$$

The Q-multiplication, used in (5), is determined by (4). The Q-inversion is defined by:

$$\frac{1}{q} = \frac{\bar{q}}{|q|^2} \quad ((6))$$

Note, that the only finite-dimensional division algebras over the real number field are the real numbers, the complex numbers and the Q-field. Functions of Q-field are defined similarly to functions of complex fields. For example:

$$\exp(q) = \exp(\alpha)[\cos(\psi) + j \sin(\psi)], \quad \psi^2 \equiv \psi_p^2, \quad j \equiv i_p \psi_p \psi^{-1}, \quad j^2 = -1 \quad ((7))$$

Using (4) - (7), we can define many Q-functions, particularly:

$$\tanh(q) = \frac{\exp(q) - \exp(-q)}{\exp(q) + \exp(-q)} = \frac{\exp(2\alpha) - \exp(-2\alpha) + 2j \sin(2\psi)}{\exp(2\alpha) + \exp(-2\alpha) + 2 \cos(2\psi)} \quad ((8))$$

Returning to (1), we now substitute into this equation Q-field (3) instead of  $\alpha$ . It gives four equations:

$$\frac{\partial \alpha}{\partial t} + k\alpha = \text{Re}\{f(\alpha + \sigma + i_q \psi_q)\} \quad ((9))$$

$$\frac{\partial \psi_p}{\partial t} + k\psi_p = \text{Im}_p\{f(\alpha + \sigma + i_q \psi_q)\}, \quad (p = 1, 2, 3) \quad ((10))$$

These equations are coupled because  $f$  is nonlinear. Thus, we got Q-modeling of the C-effects. The same scheme can be applied to equation (2) and to any nonlinear model equation. For  $f(\alpha) = \tanh(\alpha)$ , formulas (7) and (8), with shift  $\alpha \implies \alpha + \sigma$ , give explicit form for the nonlinear terms in equations (9) and (10). Note, that so-called extra-sensory effects (if they exist) can be included in this approach by assuming that  $\sigma$  has imaginary components. When  $\alpha$  and  $\sigma$  are

relatively small (in a dream or in a state of deep meditation), asymptotically (10) gives closed system of equations for  $\psi_p$ . The nonlinear terms  $\text{Im}_p\{f(i_q\psi_q)\}$  are quite different from the nonlinear term  $f(\alpha)$  for the A-process. This may explain peculiar dynamics of dreams and deep meditations, not only in content, but also in intensity.

The obtained description of the C-effects can serve for designing experiments to test the modeling. The chosen sequence of S-E-R-effects seems to be natural for the Q-modeling.

It will be interesting to apply Q-modeling to the first principles, particularly, to the problem of unification of the four major forces (gravitational, electromagnetic, weak and strong). The fundamental obstacle to the unification is that gravitation in its nature is quite different from the three other forces. It resists quantization with complex amplitudes, which is the natural description for the other forces. Perhaps, in the Q-description, gravitation can be an analog of real (automatic) process, while other three forces are more "vibrating" - analog of S-E-R-effects in the "Brain - Universe". In the Q-description "ghosts" [3,4] with the negative energy may have natural explanation.

## References

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