

The method of solving of nonlinear Schrödinger equation

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The method of solving of nonlinear Schrödinger equation is considered. Some examples of its applications are demonstrated.

Nonlinear Schrödinger equation (NLSE)

$$i \frac{\partial \phi}{\partial t} = -\frac{\partial^2 \phi}{\partial x^2} + F(|\phi|) \cdot \phi \quad (1)$$

arising in different physical context: in plasma's physics, in nonlinear optics and others [1]. Here $\phi = \phi(x, t)$.

The solution can be represented in the form

$$\phi(x, t) = y(z) \cdot \exp(i\delta \cdot t + ipz). \quad (2)$$

Here δ is the real parameter, $z = x - Vt$, $p = \frac{V}{2}$. Than from (1)

$$y_{zz} + E_0 y - F(|y|)y = 0, \quad (3)$$

where $E_0 = \frac{V^2}{4} - \delta$. The last equation we can solve by the quadratures method [2]-[3]. But we propose other method of solving (3). It look like method of solving of stationary Schrödinger equation [4].

We consider two equations

$$\begin{aligned} y'' &= (a + b \cdot y^2)y, \\ z'' &= (c + d \cdot z^2)z. \end{aligned} \quad (4)$$

We impose such condition on solutions of (4)

$$z = A \cdot \frac{y'}{y}. \quad (5)$$

Here A is some constant (or else we get discordant correlations). If we express function z through y and y' we get relationship between c and a :

$$c = -2a. \quad (6)$$

Also

$$d = 2, \quad (7)$$

and coefficients A , b can be arbitrary.

So if conditions (6)-(7) are carried out then both equations (4) are jointed with (5).

We shall demonstrate the method in two examples.

(1) The first is the next equation

$$z'' = 2 \cdot (z + z^3). \quad (8)$$

Partial solutions of this equation are known. It is

$$z_1 = \tan(x), \text{ и } z_2 = \frac{1}{\tan(x)}. \quad (9)$$

From (6) $a = -1$. We set $b = 0$ and find

$$y = A_1 \sin(x) + A_2 \cos(x). \quad (10)$$

It follows

$$z = A \cdot \frac{A_1 \cos(x) - A_2 \sin(x)}{A_1 \sin(x) + A_2 \cos(x)}. \quad (11)$$

If $A = -1$ and $A_1 = 0$ we derive from (11) the first of (9). If $A = 1$ and $A_2 = 0$ we derive from (11) the second of (9).

(2) The second is the next equation

$$z'' = -E_0 z + z^3. \quad (12)$$

We derive it from (1) where

$$F(|\phi|) = -|\phi|^2.$$

Under transformation $z \rightarrow \sqrt{2} \cdot z$ from (12) we derive

$$z'' = -E_0 z + 2z^3.$$

Then $a = \sqrt{\frac{E_0}{2}}$. Suppose that $b = 0$ we find

$$y = A_1 \exp(ax) + A_2 \exp(-ax),$$

and

$$z = A \cdot a \cdot \frac{A_1 \exp(ax) - A_2 \exp(-ax)}{A_1 \exp(ax) + A_2 \exp(-ax)}. \quad (13)$$

If $A_1 = A_2$ and $E_0 < 0$ equation (13) describe the black solitons [5].

Literature

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