COOLING OF NEUTRON STARS REVISITED: APPLICATION OF LOW ENERGY THEOREMS

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Cooling of neutrons stars proceeds mainly via neutrino emission; As an example we study the modified neutrino bremsstrahlung process $nn \rightarrow nn\nu\bar{\nu}$. The radiated energy is small compared to other scales in the system. Hence one can use low-energy theorems to compute the neutrino emissivity in terms of the non-radiative process, ie the on-shell T matrix. We find that the use of the T-matrix as compared to previous estimates based upon one pion exchange leads a substantial reduction of the predicted emissivity.

1 Introduction

The thermal evolution of neutron stars is dominated by the weak interaction and in particular neutrino interactions with the hadronic matter. The hadronic information is contained in the so-called current-current correlator $\Pi(q)$, where $q = (\vec{q}, \omega)$ is the four-momentum of the neutrinos. In general one can distinguish two different regimes:

• neutrino scattering (space-like, $\omega < q$)

• neutrino-pair emission (time-like, $\omega > q$).

In the latter case the quasi-particle response vanishes, i.e., the one-body process $n \to n + \nu + \bar{\nu}$ is forbidden by energy-momentum conservation, and the socalled URCA process

• $n \rightarrow p + e^- + \bar{\nu}$

 $p + e^- \rightarrow n + \nu$,

is forbidden unless the n, p and e^- fermi momenta satisfy the unequality $p_F^p + p_F^e > p_F^n$, which is very unlikely.

Therefore the bremsstrahlung process can only take place in the presence of spectator nucleon (referred to as modified processes), e.g.:

• $n + N \rightarrow n + N + \nu + \overline{\nu} \ (N = n, p)$

• $n + n \rightarrow n + p + e^- + \bar{\nu}$ (+ inverse)

In the pioneering work of Friman and Maxwell² these processes were computed in the extreme soft neutrino limit in Born approximation with the NN interaction presentated by a Landau type interaction plus a one-pion exchange part.

With respect to this approach several questions can be asked; how accurate is



Figure 1: Leading order diagrams for soft bremsstrahlung

the use of Born approximation, what is the contribution of other mesons, and how important are relativistic effects?

It is the aim of the present study to address these issues by computing the emissivity using a low-energy theorem on the basis of the observation that the neutrino radiation is very soft compared to other scales in the system $(\omega \approx T = 1 \text{ MeV})$; this allows one to use an empirical T-matrix, fully determined by phase shifts.

2 Soft electro-weak bremsstrahlung

First we consider the "Soft-photon" amplitude in free space

The original soft-photon theorem states that the first two terms in an expansion of the electromagnetic bremsstrahlungs amplitude in photon four-momentum q are fully determined by the amplitudes of the non-radiative process, ¹

$$M = A/\omega + B + O(\omega)$$

This result can be generalized in several ways, e.g. to virtual bremsstrahlung, $(q^2 > 0)$ and also to the case of the weak axial vector current. Here we shall restrict ourselves to the leading order, the A term, in which case only radiation off external legs contributes (see Fig. 1), and the amplitude can de expressed as

$$M_{\mu} = TS(p_1 - q)\Gamma_{\mu} + \Gamma_{\mu}S(p_1' + q)T + \{1 \leftrightarrow 2\}$$

The weak vector and axial vector vertices Γ are given by low-energy neutral current Hamiltonian,

$$H = \frac{G_F}{2\sqrt{2}} B_\mu \bar{\psi} \gamma_\mu (1 - \gamma_5) \psi$$

with the hadron current given by $B^{\mu} = \sum_{i=n,p} \bar{\psi}_i \gamma^{\mu} (C_{v,i} - \gamma_5 C_{A,i}) \psi_i$. Hence in the non-relativistic limit

$$\Gamma_{\mu}(\text{vector}) \approx g_V \gamma_{\mu} \rightarrow \delta_{\mu,0}$$

$$\Gamma^i_\mu(\text{axial}) \approx g^i_A \gamma_5 \gamma_\mu \to g^i_A \vec{\sigma} \ (i=p,n; \ g^p_A = -g^n_A)$$

The two Feynman propagators (corresponding to prior and post emission) are given by

$$S(p\pm q) = \frac{1}{\gamma(p\pm q) - m} \approx \frac{\pm m}{p.q} = \frac{\pm 1}{\omega - \vec{p}.\vec{q}/m} \approx \pm \frac{1}{\omega} (1 + O(p.q/m\omega)).$$

Hence in this order the amplitude M can be expressed as a commutator of T with the space component of the axial weak current operator

$$M_i = \frac{g_A}{\omega} [T, S_i]$$

where $S = (\sigma_1 + \sigma_2)/2$, and in this order there is no vector contribution. Which terms survive the commutator $[T, \Gamma^{\text{weak}}]$? Looking at the structure of the NN amplitude given in the appendix one sees that there will be nonvanishing contributions from tensor, spin-orbit and quadratic spin-orbit terms in T. Thus one may conclude that only nn spin correlations contribute to the emissivity. However, we note that in the past the effective nn interactions were restricted to purely local interactions and hence the spin-orbit interactions were not considered. As an illustation we compute the cross section for NN electroweak bremsstrahlung in free space as a function of the relative nn momentum (in the nn cm system). In fig. 2a we show the results for full T- matrix and its separate components. In fig. 2b the cross section for the full T-matrix is compared with the result for π , $\pi + \rho$ and also σ meson exchange.

We note that the OPE exchange (which forms the basis of most standard "cooling scenarios" overestimates the results obtained with the full T-matrix by about a factor 5 for relative momenta in the range appropriate for neutron matter at normal density ($p_F = 1.3 \text{fm}^{-1}$) (a similar result has been found in ⁴). It is also seen that the contribution from the OPE tensor force is largely canceled by that from ρ exchange, and that the sum of OPE and ORE is much closer to the T -matrix result than OPE .

3 The np case

The np case is slightly different, since the axial vector couplings of neutron and proton have opposite values $g_A^p = -g_A^n = -1.25$. Hence in this case the net result can be expressed as the sum of a commutator and an anticommutator, where the latter corresponds to the exchange diagrams

$$M_i = \frac{g_A}{\omega}([T^{dir}, D_i] + \{T^{exch}, D_i\})$$



Figure 2: Cross section for weak nn bremsstrahlung for $\omega = 1$ MeV as a function of the relative momentum $p = (p_1 - p_2)/2$; top: full *T*-matrix (solid line), tensor+spin-spin (dashed), spin-orbit (dotted) and quadratic spin-orbit (dashed-dotted line); bottom: OPE (solid), OPRE (dotted), OPRSE (long dashed), direct OPE (short dashed), direct OPRE (dashed-doubly dotted) and full *T* (dashed-dotted line).

where $D = (\sigma_1 - \sigma_2)/2$, and $T^{dir,exch}$ are defined in the appendix.

In this case also the so-called Landau terms in the *T*-matrix, of the form $g\sigma_1.\sigma_2$ and $g'\sigma_1.\sigma_2\tau_1.\tau_2$ contribute to the anti-commutator. In practice the proton fraction in a neutron star is quite small and therefore we find that the relative contribution from np is quite unimportant.

4 Emissivity in medium

The emissivity in the medium with given density and temperature T can be computed from the amplitude in two ways: (i) via the (imaginary part) of the current-current correlator, or (ii) using the Fermi golden rule. The former approach is more general, but the latter is simpler to use and is sufficient in the present case. In lowest order in the virial expansion it amounts to convoluting the free space matrix elements $|M^2|$ with the hadronic Fermi-Dirac functions

$$\epsilon = \prod_i \int d^3 p_i \frac{d^4 q_1}{\omega_1} \frac{d^4 q_2}{\omega_2} (\omega_1 + \omega_2) \delta^4 (p_1 + p_2 - p_3 - p_4 - q_1 - q_2) F| < M_j > |^2,$$

where $F = f_1 f_2 (1-f_3)(1-f_4)$. In practice the integrals are simplified by taking the absolute values of the nucleon momenta equal to the fermi momenta. If we consider the ratio of the emissivity calculated with the full T matrix and the one from the OPE² we obtain about a factor 4-5 reduction. This is inline with the conclusion in ⁴ for pure neutron matter.

Other medium that need to be considered in further work are 1) replacement of the free T- by a G-matrix, which takes into account the Pauli blocking and other medium effects, 2) medium renormalization of the axial coupling vertex, 3) inclusion of higher order medium effects such as dressing Green functions. In the latter case one has to take care to conserve the symmetries (CVC, PCAC) of the problem (which are conserved in the present case).

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Appendix: Structure of the on-shell NN amplitude

The on-shell T-matrix is fully determined by the nn phaseshifts. First we consider nn; in a covariant approach one has

$$T^{on} = T_{\alpha}(s, t, u)\bar{u}(p_2)\Omega_{\alpha}u(p_2) \times \bar{u}(p_1)\Omega_{\alpha}u(p_1)$$

$$\Omega_{\alpha} = \{1, \sigma_{\mu,\nu}, \gamma_{\mu}, \gamma_5, \gamma_{\mu}\gamma_5\}$$

Non-relativistically

$$T = T_c + T_s \sigma_1 \sigma_2 + T_T S_{12} + T_{SO} L S + T_Q Q_{12}$$

In case of np there are twice as many components which can be distinguished by isospin I = 0, 1, i.e. $T^{dir} = (T^0 + T^1)/2$ and $T^{exch} = (T^0 - T^1)/2$.

References

- 1. F.E. Low, Phys. Rev. 110,974(1958)
- 2. B.L. Friman and O.V. Maxwell, ApJ. 232,541(1979)
- 3. R. Timmermans *et al*, in preparation
- 4. C. Hanhart, D. Phillips and S. Reddy, astro-ph/0003445

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