## The Origin of the Difference between Multiplicities in  $e^+e^-$  Annihilation and Heavy Ion Collisions.

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## Abstract

Multiplicities in  $e^+e^-$  annihilation and relativistic heavy ion collisions show remarkable similarities at high energies. A thermal-statistical model is proposed to explain the differences which occur mainly at low beam energies. Two different calculations are performed, one using an approximate thermodynamic relationship, the other using a full thermal model code. The results are in qualitative agreement, suggesting that the interplay of baryon density and temperature tends to systematically suppress the total multiplicity at lower beam energies.

Recently, the PHOBOS experiment at RHIC has shown results on the total charged particle multiplicity produced in heavy ion collisions over a wide range of collision energies as well as the collision centrality, characterized by the number of participating nucleons [1]. Three surprising connections emerge from comparison of this data with particle multiplicities measured in elementary collisions (pp,  $\overline{p}p$ , and  $e^+e^-$ ):

- (1) The total number of primary charged particles produced in A+A reactions  $(N_{ch}^{A+A})$  scales linearly with the number of participants  $(N_{part})$ .
- (2) The multiplicity per participant pair in A+A  $(N_{ch}^{A+A} / \langle N_{part}/2 \rangle)$  agrees with that measured in  $e^+e^ (N_{ch}^{e^+e^-})$  within 10% over a large range in  $\sqrt{s}$ and  $\sqrt{s_{NN}}$ . In general, the  $e^+e^-$  data has not fully removed the contribution from weak decays, but this is generally less than a 10% correction.
- (3) This agreement is not simply in the total multiplicity, but extends over the full rapidity range (relative to the thrust axis in the  $e^+e^-$  case).

This agreement in total multiplicity suggests a certain universality in particle production, a result consistent with the thermal-statistical approach started by Fermi, Landau and Hagedorn [2,3,4]. However, the same data set shows a systematic deviation between the multiplicities below  $\sqrt{s_{NN}} = 30$  GeV, and increasing as the energy gets lower. It is the purpose of this letter to explore a simple physics hypothesis which can explain this deviation in a semi-quantitative fashion. To compare multiplicities between  $e^+e^-$  and heavy ion collisions we introduce the quantity

$$
\Delta N_{ch} \equiv N_{e^+e^-} - \frac{N_{AA}}{N_{part}/2} \tag{1}
$$

where, in an obvious notation,  $N_{part}$  denotes the number of participants in A+A collisions, while  $N_{e^+e^-}$  is the total multiplicity in an  $e^+e^-$  collision, and  $N_{AA}$  is the total multiplicity in an A+A collision. It is proposed that this difference is directly related to the thermodynamic variables determined from the total multiplicities measured in  $A+A$  collisions, namely the freezeout values of the temperature, T, and baryon chemical potential  $\mu_B$ . For the multiplicity of charged particles this relation reduces to

$$
\Delta N_{ch} = \frac{\mu_B}{3T} \tag{2}
$$

Various improvements on this relation will be discussed below.

Examination of the yields of different particle species over a wide range in  $\sqrt{s_{NN}}$  shows a large variation of the baryon to pion ratio  $(p/\pi)$ . Conversely,  $p + p$  collisions show a somewhat more rapid dependence of this ratio with beam energy. This can be understood heuristically by saying that relative to proton-proton collisions, collisions involving nuclei  $(p+A, A+A)$  are distinguished by the larger "stopping power" of the nuclear targets. While this phenomenon is not well-understood theoretically, it has been characterized phenomenologically in several different ways. In proton-nucleus collisions, the concept of "rapidity loss" is usually used to measure how much energy the proton projectile loses in the multiple collisions in the nuclear target [5]. While the data are not trivial to interpret, given the state of understanding of longitudinal dynamics, one generally finds a substantial net-baryon density near mid-rapidity in A+A collisions, higher than  $p+p$  and  $e^+e^-$  reactions.

In Au+Au collisions, it is often postulated that the large particle multiplicities create a thermally and chemically equilibrated system. This suggests using statistical models to characterize the relative population of hadronic states [6,7]. In these models, the main parameters are the freeze-out temperature  $(T)$ , which is associated with the energy density of the system, and the baryochemical potential  $(\mu_B)$ , which is directly related to the density of the net baryon number distributed in the freeze-out volume  $(N(p) - N(\overline{p}) \propto e^{\mu_B/T} - e^{-\mu_B/T})$ . It is less clear why similar fits should work for elementary collisions (e.g.  $pp$ )

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and  $e^+e^-$ ). However, the work of Becattini has shown that statistical models prove to be an equally useful tool in describing the relative yields of hadrons in collisions with relatively small multiplicities [8], although additional care must be taken to guarantee appropriate conservation of quantum numbers (e.g. strangeness and baryon number).

Thermal fits made by a number of authors [6,7] show that increasing the  $\sqrt{s_{NN}}$ in A+A collisions leads to an increase in T and a correlated decrease in  $\mu_B$ , shown in Fig. 1. This has been interpreted by Cleymans and Redlich by postulating a fixed relationship of the freezeout parameters, such that  $\langle E \rangle / \langle N \rangle \sim 1$ GeV [9]. Whatever the physical scenario implied by this condition, it provides a useful way to determine these parameters as a function of beam energy, and to interpolate between available data points. However, it turns out that this criterion (called "Thermal I") does not perfectly describe the existing data. A somewhat better description, although purely phenomenological, can be made by a sixth-order polynomial fit in  $\mu_B$  to the same data in the  $(T,\mu_B)$  plane ("Thermal II")  $[10]$ :

$$
T(\mu_B) = 0.16446 - 0.11196\mu_B^2 - 0.139139\mu_B^4 + 0.0684637\mu_B^6
$$

In this work, we will show both parametrizations where possible.

Also in this work, we use a parametrization of  $\mu_B$  as a function of  $\sqrt{s}$  made by the authors in Ref. [11]

$$
\mu_B(\sqrt{s}) = \frac{1.2735}{(1 + 0.2576\sqrt{s})} \tag{3}
$$

To apply this information to the heavy ion and  $e^+e^-$  data, we will invoke a simple thermodynamic condition. When dealing with blackbody radiation, one typically sets the Gibbs potential  $G = E - TS + pV = \sum_i \mu_i N_i \sim \mu_B N_B$ , since the other chemical potentials (e.g. strangeness, charge, isospin) are usually smaller than the baryochemical potential. In this formula,  $E$  is the internal energy, T is the temperature, S the entropy, p the pressure,  $\mu_B$  the baryochemical potential and  $N_B$  the baryon number which must be conserved in the interaction. This expression can be rearranged to show how the entropy is related to the other variables:

$$
S = \frac{(E + pV) - \mu_B N_B}{T} = S_0 - S_B \tag{4}
$$

where

$$
S_0 = \frac{E + pV}{T} \tag{5}
$$

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is the entropy due to the internal energy and the pressure of the system, while

$$
S_B = \frac{\mu_B N_B}{T} \tag{6}
$$

is interpreted as the entropy bound up in the conserved baryons, suppressing the total entropy.

The  $(E + pV)/T$  term can be understood as the one that controls particle production in the absence of conserved baryon charges (i.e.  $\mu_B = 0$ ). It is assumed that this is universal for all strongly interacting collision systems with the same expansion features, most importantly the dominance of 1D expansion in the early stages. The second term is thus a correction which will only be important when  $\mu_B N_B/T$  is non-negligible, i.e. at large  $\mu_B$  or small T or both.

This correction to the total entropy can be estimated in a crude way as follows:

- A factor of  $\alpha = 4$  to normalize entropy to the number of particles (as is relevant for a massless Boltzmann gas).
- A factor of  $N_{part}/2$  to give the total change in multiplicity per participant baryon pair. This cancels the  $N_B$  in the numerator since it is precisely the number of participants which determines the conserved baryon number.
- A factor of  $\beta = 3/2$  which accounts for unmeasured neutral pions. This is based on the assumption that we are calculating the entropy of the lighter pions that would have been produced except for the non-zero  $\mu_B$  enforcing the presence of heavy baryons.

In other words, this scenario postulates  $S/N_{ch} = 6$ . Dividing by all these factors gives:

$$
\Delta N_{ch} = \frac{2}{\alpha \beta N_{part}} \frac{\mu_B N_B}{T}
$$
  
=  $\frac{2}{\alpha \beta} \times \frac{N_B}{N_{part}} \times \frac{\mu_B}{T}$   
=  $\frac{\mu_B}{3T}$  (7)

This is the *multiplicity* that must be added to the low-energy results at a given  $\sqrt{s}$  to account for the entropy that would have been available except for the need to conserve baryon charge.

It turns out that direct calculations with thermal models [12], give the result that the entropy divided by the multiplicity of final-state charged particles (after strong decays) is  $S/N_{ch} = 7.2$ . This number should be compared to  $\alpha\beta =$ 

4

6 from the considerations above. For the subsequent calculations, the more theoretically relevant number will be used instead of the simpler estimate. The difference between them should be seen as contributing to an overall theoretical uncertainty.

When this is done, we get the results shown in Fig. 2, where the multiplicities have been divided by the Landau-Fermi expression  $(N_{ch} = 2.2s^{1/4})$  [2,3,13]. The  $e^+e^-$  results are shown as open squares, the original A+A results are shown as open circles, and the " $\mu_B$ "-corrected A+A results are shown as closed circles. It is surprising that this simple model works as well as it does, since it is nothing more than correcting for the fact that the initial baryons must be present in the final state, and thus take up energy that would have normally gone to normal thermal particle production (mainly pions).

However, it can be argued that the  $4\pi$  multiplicity collisions has an extra component that would not be found in  $e^+e^-$  collisions, namely the participant baryons themselves. To correct for these, we make two additional transformations on the A+A data.

- " $m_P$ ": Subtract 2 $m_P$  from  $\sqrt{s}$  to correct for the difference in the mass of the beam particle
- " $n_B$ ": Subtract 1, assuming that the net baryons will be either p or n equally.

These corrections are shown to have relatively little effect on the basic result, as seen by the thick dotted line in Fig. 3. Thus, it is not possible to distinguish their relevance by comparison with the  $e^+e^-$  data. It should also be noted that just applying the  $m_P$  and  $n_B$  corrections, without the  $\mu_B$  correction, has little effect on the initial result as well, as shown in Fig. 3.

A more quantitative check of the physical picture discussed here can be made by calculating the entropy density vs.  $\mu_B$  in a full statistical-thermal model calculation [12,14]. The freezeout contour is the same as in the calculation above. To relate this curve to experimental data, one considers the translation of the experimental variable  $N_{ch}^{\hat{A}+A}/(N_{ch}^{e^+e^-}N_{part}/2)$  to parameters accessible in thermal models. From the fits performed on  $p+p$  and  $e^+e^-$  data in Ref. [8], we find that the  $T$  parameter is essentially constant over a wide range of energy  $T = T_0 \sim 170$  MeV. In this case,  $s(T, \mu_B) = s(T_0, 0)$  and thus should be constant with  $\sqrt{s}$ . It also appears to be the asymptotic value reached in ultra-high energy  $A + A$  collisions, if current trends are to be believed. In this case, we assume that

$$
N_{ch}^{e^+e^-} = C^{e^+e^-} V^{e^+e^-} s_0 \tag{8}
$$

where  $V^{e^+e^-}$  is the freezeout volume and  $C_{e^+e^-}$  is a constant relating the number of detected particles to the total entropy (again, approximately 4).

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In this picture, the energy dependence of the total multiplicity is determined dominantly by the freezeout volume.

In A+A collisions, the multiplicity has been found to scale linearly with the number of participating nucleons  $N_{part}$ . Since the volume of the initial nuclei scales with  $A$ , and the freezeout entropy scales with  $V$ , this suggests that  $V \propto N_{part}/2 = V^{A+A} N_{part}/2$ . We then assume that

$$
N_{ch}^{A+A} = \frac{N_{part}}{2} C^{A+A} V^{A+A} s(T, \mu_B)
$$
\n
$$
(9)
$$

where  $V_{A+A}$  is the effective volume per participant pair.

With these assumptions, the ratio shown in Figure 3 is

$$
\frac{2}{N_{part}} \frac{N_{ch}^{A+A}}{N_{ch}^{e^+e^-}} = \frac{C^{A+A}}{C^{e^+e^-}} \frac{V^{A+A}}{V^{e^+e^-}} \frac{s(T, \mu_B)}{s_0}.
$$
\n(10)

The right-hand side of this equation has two sets of constants (C and V) and one ratio that depends on beam energy. The constants C control the proportionality between the total entropy and the total charged particle multiplicity. Landau and Belenkij [15] argued that this constant does not depend on the system size, so it makes sense to set this ratio to unity. This presumption is supported by the overall similarity in the particle production [16] (although strangeness is clearly suppressed in the smaller systems). They also did not think there would be a change in the relation of the multiplicity to the entropy as a function of beam energy or initial baryon density. The ratio of the volumes might not be expected to be the same. However, the agreement (to the 10% level) of the total multiplicity per participant pair in A+A and the total multiplicity in  $e^+e^-$  suggests that these volumes are similar, even as a function of energy.

In Fig. 2 we compare the ratio  $s(T, \mu_B)/s_0$ , shown for the two parametrizations of  $T(\mu_B)$  (Thermal I and Thermal II), with the ratios discussed previously. The agreement between the thermal model curve and the  $A+A$  data is in reasonable qualitative agreement. This suggests that the previously-made physics assumption

$$
\frac{C^{A+A}}{C^{e+e}} \frac{V^{A+A}}{V^{e+e-}} = 1\tag{11}
$$

is a reasonable one.

It should be noted that the concept of "pion suppression" was previously discussed by Gazdzicki et al [17] by reference to data from a similar set of

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experiments as discussed in [1]. However, the phenomenon discussed in this work is concerned with the suppression of the total entropy rather than just the pions, and is a feature which arises naturally in the context of thermal models. The models here also include all available meson and baryon resonances, as opposed to just delta resonances.

To make the relative energy dependence of mesons and baryons clearer, we show their respective contributions to the entropy density as a function of beam energy in Fig. 4, again for two parametrizations. The individual and total entropy densities are both divided by  $T^3$  to remove the expected temperature dependence. They are then multiplied by a factor if  $\pi^2/4$ , which transforms the quantity  $s/T^3$  into the effective number of degrees of freedom  $n(T)$  of a massless Boltzmann gas. The main result is that the baryon contribution completely dominates at low energies, but the mesons are equal at  $\sqrt{s_{NN}} \sim 10$ GeV and their contribution exceeds that of the baryons by a factor of  $\sim 2$ and saturates. However, as was also noted in Ref. [18], it is observed that the quantity  $s/T^3$  is constant over a large range in center of mass energies (down to  $\sqrt{s_{NN}}$  = 5 GeV) and diverges only at very low energies (presumably due to the associated rapid decrease in  $T$ ). This result shows that not only are the number of degrees of freedom similar at freezeout for  $A + A$  and  $e^+e^-$ , but they are similar for heavy ion collisions over a large range of collision energies. From this result, that freezeout occurs for  $s/T^3 = const.$ , one might also explain the suppression of the entropy density  $s = const. \times T^3$ , as due to the lower temperatures associated with larger  $\mu_B$ . In either case, the suppression of the total multiplicity results from the non-trivial interplay between  $\mu_B$  and T.

In conclusion, the difference between the charged particle multiplicity per participant pair in A+A and the multiplicity in  $e^+e^-$  can be explained by the suppression of entropy due to the presence of a conserved quantum number, manifest as the net-baryon density. A semi-quantitative understanding of the existing data has been achieved both by simple thermodynamic arguments as well as more detailed thermal model comparisons.

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Fig. 1. Compilation of thermal-statistical model fits to heavy ion data over a wide range of beam energies. Two parameterizations of this data are shown, one based on  $E/N \sim 1$  (solid line) and the other a sixth-order polynomial fit (dotted line).

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Fig. 2. Ratio of  $N_{ch}$  for  $e^+e^-$  annihilation (open squares) and  $N_{ch}/\langle N_{part}/2 \rangle$  for A+A collisions (open circles) to a Landau-hydro expression for the total multiplicity  $(2.2s^{1/4})$ . Also shown are the same ratios for A+A after correcting by  $\mu_B/(\alpha\beta/2)T$ (closed circles). The ratio of the entropy density to the asymptotic value as a function of  $\sqrt{s}(\mu_B)$  is shown for the two paths through the T,  $\mu_B$  plane shown in Fig. 1. These should be compared with the open circles, as it already contains the effects described in the main text.



Fig. 3. Ratio of  $N_{ch}$  for  $e^+e^-$  annihilation (open squares) and  $N_{ch}/\langle N_{part}/2 \rangle$  for  $A+A$  collisions (open circles) to a Landau-hydro expression for the total multiplicity  $(2.2s^{1/4})$ . Also shown are the same ratios for A+A after various corrections: 1)  $\mu_B/3T,$   $n_B,$   $2m_P$  corrections (dot-dashed line) and 2)  $n_B,$   $2m_P$  corrections (dot-dashed line).

![](_page_11_Figure_0.jpeg)

Fig. 4. Entropy density normalized by  $T^3$  and scaled by  $\pi^2/4$ , shown separately for mesons and baryons as a function of  $\sqrt{s}(\mu_B)$ . For each species, the two parametrizations of  $T(\mu_B)$  are shown, giving an estimate of the systematic uncertainty. While the two components exchange their relatve dominance as a function of beam energy, their total is constant over most of the calculated energies.