COLOR SUPERCONDUCTIVITY IN DENSE QUARK MATTER

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Abstract

A brief introduction into the properties of dense quark matter is given. Recently proposed gapless color superconducting phases of neutral and beta-equilibrated dense quark matter are discussed. The current status in the field is described, and the promising directions of the future research are outlined.

1 Introduction

At sufficiently high baryon density, matter is expected to be deconfined. The physical degrees of freedom in a deconfined phase are quarks and gluons, rather than usual hadrons. At present the theory cannot predict reliably where in the QCD phase diagram the corresponding deconfinement transition should occur. The issue gets further complicated by the fact that the deconfinement is not associated with a symmetry-related order parameter and, thus, does not need to be marked by any real phase transition. Leaving aside this well-known conceptual difficulty, here I discuss the recent progress in the studies of cold and dense matter which, owing in part to the property of the asymptotic freedom in QCD, allows a relatively rigorous treatment.

It was suggested long time ago that quark matter may exist inside the central regions of compact stars [1]. By making use of the property of asymptotic freedom in QCD [2], it was argued that quarks interact weakly, and that realistic calculations taking full account of strong interactions are possible for sufficiently dense matter [3]. The argument of Ref. [3] consisted of the two main points: (i) the long-range QCD interactions are screened in dense medium causing no infrared problems, and (ii) at short distances, the interaction is weak enough to allow the use of the perturbation theory. As will become clear below, the real situation in dense quark matter is slightly more subtle.

2 Color superconductivity

By assuming that very dense matter is made of weakly interacting quarks, one could try to understand the thermodynamic properties of the corresponding ground state by first completely neglecting the interaction between quarks. In order to construct the ground state, it is important to keep in mind that quarks are fermions, i.e., particles with a half-integer spin, $s = 1/2$. They should obey the Pauli exclusion principle which prohibits for two identical fermions to occupy the same quantum state.

In the ground state of non-interacting quark matter at zero temperature, quarks occupy all available quantum states with the lowest possible energies. This is formally described by the following quark distribution function:

$$f_F(\mathbf{k}) = \theta\left(\mu - E_{\mathbf{k}}\right), \quad \text{at} \quad T = 0, \tag{1}$$

where μ is the quark chemical potential, and $E_{\mathbf{k}} \equiv \sqrt{k^2 + m^2}$ is the energy of a free quark (with mass m) in the quantum state with the momentum $\mathbf{k}$ (by definition, $k \equiv |\mathbf{k}|$). As one can see, $f_F(\mathbf{k}) = 1$ for the states with $k < k_F \equiv \sqrt{\mu^2 - m^2}$, indicating that all states with the momenta less than the Fermi momentum k_F are occupied. The states with the momenta greater than the Fermi momentum k_F are empty, i.e., $f_F(\mathbf{k}) = 0$ for $k > k_F$.

It appears that the perturbative ground state of quark matter, characterized by the distribution function in Eq. (1), is unstable when there is an attractive (even arbitrarily weak in magnitude!) interaction

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between quarks. This is because of the famous Cooper instability [4]. The instability develops as a result of the formation of Cooper pairs $\langle q_{\mathbf{k}} q_{-\mathbf{k}} \rangle$ made of quarks from around the highly degenerate Fermi surface, i.e., quarks with the absolute value of momenta $k \simeq k_F$. Such Cooper pairs are bosonic states, and they occupy the same lowest energy quantum state at $T = 0$, producing a version of a Bose condensate. The corresponding ground state of quark matter is then a superconductor. This is similar to the ground state of an electron system in the Bardeen-Cooper-Schrieffer (BCS) theory of low-temperature superconductivity [5]. Of course, some qualitative differences also arise because quarks, unlike electrons, come in various flavors (e.g., up, down and strange) and carry non-Abelian color charges. To emphasize the difference, superconductivity in quark matter is called *color superconductivity*. For recent review on color superconductivity see Ref. [6].

As in low-temperature superconductors in solid state physics, one of the main consequences of color superconductivity in dense quark matter is the appearance of a nonzero gap in the one-particle energy spectrum. In the simplest case, the dispersion relation of gapped quasiparticles is given by

$$E_\Delta(k) = \sqrt{(E_{\mathbf{k}} - \mu)^2 + \Delta^2}, \tag{2}$$

where Δ is the gap. The presence of a nonzero gap affects kinetic (e.g., conductivities and viscosities) as well as thermodynamic (e.g., the specific heat and the equation of state) properties of quark matter [6].

Historically, it has been known for a rather long time that dense quark matter should be a color superconductor [7, 8]. In many studies in the past this fact was commonly ignored, however. Only recently, the potential importance of this phenomenon was appreciated. To large extent, this has been triggered by the observation [9] that the value of the color superconducting gap Δ can be as large as $100 \, \text{MeV}$ at baryon densities existing in the central regions of compact stars, i.e., at densities which are a few times larger than the normal nuclear density, $n_0 \simeq 0.15 \, \text{fm}^{-3}$. A posteriori, of course, this estimate is hardly surprising within the framework of QCD, in which the energy scale is set by $\Lambda_{\text{QCD}} \simeq 200 \, \text{MeV}$. Yet this observation was very important because the presence of a large energy gap in the quasiparticle spectrum may allow to extract signatures of color superconducting matter in observational data from compact stars.

3 Two-flavor color superconductivity ($N_f = 2$)

The simplest color superconducting phase is the two-flavor color superconductor (2SC). This is a color superconducting phase in quark matter made of up and down quarks.

In weakly interacting regime of QCD at asymptotic densities, the 2SC phase of matter was studied from first principles in Ref. [10]. It should be mentioned, however, even at the highest densities existing in the central regions of compact stars ($n \lesssim 10n_0$) quark matter is unlikely to be truly weakly interacting. In such a situation, the use of the microscopic theory of strong interactions is very limited, and one has to rely on various effective models of QCD. A very simple type of such a model, used for the description of color superconducting matter, is the Nambu-Jona-Lasinio (NJL) model with a local four-fermion interaction (for a review see, e.g., Ref. [11]). One of its simplest versions is defined by the following Lagrangian density [12]:

$$\begin{aligned}
\mathcal{L}_{\text{NJL}} &= \bar{\psi}_i^a \left(i\gamma^\mu \partial_\mu + \gamma^0 \mu - m_i^{(0)} \right) \psi_i^a + G_S \left[(\bar{\psi}\psi)^2 + (i\bar{\psi}\gamma_5\vec{\tau}\psi)^2 \right] \\
&+ G_D (i\bar{\psi}^C \varepsilon \epsilon^a \gamma_5 \psi)(i\bar{\psi}\varepsilon\epsilon^a \gamma_5 \psi^C),
\end{aligned} \tag{3}$$

where $\psi^C = C\bar{\psi}^T$ is the charge-conjugate spinor and $C = i\gamma^2\gamma^0$ is the charge conjugation matrix. The matrix C is defined so that $C\gamma_\mu C^{-1} = -\gamma_\mu^T$. Regarding the other notation, $\vec{\tau} = (\tau^1, \tau^2, \tau^3)$ are the Pauli matrices in the flavor space, while $(\varepsilon)^{ik} \equiv \varepsilon^{ik}$ and $(\epsilon^a)^{bc} \equiv \epsilon^{abc}$ are the antisymmetric tensors in the flavor and in the color spaces, respectively. The dimensionful coupling constant $G_S = 5.01 \, \text{GeV}^{-2}$ and the momentum integration cutoff parameter $\Lambda = 0.65 \, \text{GeV}$ (which appears only in loop calculations) are

adjusted so that the values of the pion decay constant and the value of the chiral condensate take their standard values in vacuum QCD: $F_\pi = 93$ MeV and $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = (-250\,\text{MeV})^3$ [12]. Without loss of generality, the strength of the coupling constant G_D is taken to be proportional to the value of G_S: $G_D = \eta G_S$ where η is a dimensionless parameter of order 1. It is important that η is positive, which corresponds to an attraction in the color-antisymmetric diquark channel. This property is suggested by the microscopic interaction in QCD at high density, as well as by the instanton-induced interaction at low density [9].

The color-flavor structure of the condensate of spin-0 Cooper pairs in the 2SC phase reads

$$\left\langle \left(\bar{\psi}^C \right)_i^a \gamma^5 \psi_j^b \right\rangle \sim \varepsilon_{ij} \epsilon^{abc}. \tag{4}$$

In a fixed gauge, the color orientation of this condensate can be chosen arbitrarily. It is conventional to point the condensate in the third (blue) color direction, $\left\langle \left(\bar{\psi}^C \right)_i^a \gamma^5 \psi_j^b \right\rangle \sim \varepsilon_{ij} \epsilon^{ab3}$. Then, the Cooper pairs in the 2SC phase are made of the red and green quarks only, while blue quarks do not participate in pairing at all. These unpaired blue quarks give rise to ungapped quasiparticles in the low-energy spectrum of the theory.

The flavor antisymmetric structure in Eq. (4) corresponds to a singlet representation of the global $SU(2)_L \times SU(2)_R$ chiral group. This means that the (approximate) chiral symmetry is not broken in the 2SC ground state. In fact, there are no other global continuous symmetries which are broken in the 2SC phase. There exist, however, several approximate symmetries which are broken. One of them is the approximate $U(1)_A$ symmetry which is a good symmetry at high density when the instantons are screened [13]. Its breaking in the 2SC phase results in a pseudo-Nambu-Goldstone boson [14]. Additional four pseudo-Nambu-Goldstone states may appear as a result of a less obvious approximate axial color symmetry discussed in Ref. [15].

In the ground state, the vector-like $SU(3)_c$ color gauge group is broken down to the $SU(2)_c$ subgroup. Therefore, five out of total eight gluons of $SU(3)_c$ become massive due to the Anderson-Higgs mechanism. The other three gluons, which correspond to the unbroken $SU(2)_c$, do not interact with the gapless blue quasiparticles. They give rise to low-energy $SU(2)_c$ gluodynamics. The red and green quasiparticles decouple from this low-energy $SU(2)_c$ gluodynamics because they are gapped [16].

The gap equation in the NJL model in the mean field approximation looks as follows:

$$\Delta \simeq \frac{4G_D}{\pi^2} \int_0^\Lambda \left(\frac{\Delta}{\sqrt{(p-\mu)^2 + \Delta^2}} + \frac{\Delta}{\sqrt{(p+\mu)^2 + \Delta^2}} \right) p^2 dp. \tag{5}$$

This gap equation is analogous to the Schwinger-Dyson equation in QCD [10] in which the gluon long-range interaction is replaced by a local interaction.

The approximate solution to the gap equation in Eq. (5) reads

$$\Delta \simeq 2\sqrt{\Lambda^2 - \mu^2} \exp \left(-\frac{\pi^2}{8G_D\mu^2} + \frac{\Lambda^2 - 3\mu^2}{2\mu^2} \right). \tag{6}$$

This is very similar to the BCS solution in the case of low temperature superconductivity in solid state physics [5]. As in the BCS theory, it has the same type non-analytic dependence on the coupling constant and the same type dependence on the density of quasiparticle states at the Fermi surface. (Note that in QCD at asymptotic density, in contrast, the long-range interaction leads to a qualitatively different non-analytic dependence of the gap on the coupling constant, $\Delta \sim \mu \, \alpha_s^{-5/2} \exp \left(-C/\sqrt{\alpha_s} \right)$ where $C = 3(\pi/2)^{3/2}$ [10]).

When the quark chemical potential μ takes a value in the range between $400\,\text{MeV}$ and $500\,\text{MeV}$, and the strength of the diquark pairing is $G_D = \eta G_S$ with η between 0.7 and 1, the value of the gap appears to be of order $100\,\text{MeV}$. In essence, this is the result that was obtained in Ref. [9].

4 Color-flavor locked phase ($N_f = 3$)

It may happen that dense baryonic matter is made not only of the lightest up and down quarks, but of strange quarks as well. In fact, because of a possible reduction in the free energy from converting non-strange quarks into strange quarks, one may even speculate that strange quark matter is the true ground state of baryonic matter [17].

The constituent strange quark mass in vacuum QCD is estimated to be of order $500\,\text{MeV}$. Its current mass is about $100\,\text{MeV}$. In dense baryonic matter in stars, therefore, the strange quark mass should be somewhere in the range between the two limits, $100\,\text{MeV}$ and $500\,\text{MeV}$. It is possible then that strange quarks also participate in Cooper pairing.

Let me first discuss an idealized version of three-flavor quark matter, in which all quarks are assumed to be massless. A more realistic case of a nonzero strange quark mass will be discussed briefly in Secs. 6 and 7. In the massless case, the quark model possesses the global $\text{SU}(3)_L \times \text{SU}(3)_R$ chiral symmetry and the global $\text{U}(1)_B$ symmetry connected with the baryon number conservation. This is in addition to $\text{SU}(3)_c$ color gauge symmetry. Note that the generator $Q = \text{diag}_{\text{flavor}}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ of the $\text{U}(1)_{\text{em}}$ symmetry of electromagnetism is traceless, and therefore it coincides with one of the vector-like generators of the $\text{SU}(3)_L \times \text{SU}(3)_R$ chiral group.

To large extent, the color and flavor structure of the spin-0 diquark condensate of Cooper pairs in the three-flavor quark matter is fixed by the symmetry of the attractive diquark channel and the Pauli exclusion principle. In particular, this is given by the following ground state expectation value [18]:

$$\left\langle \left(\bar{\psi}^C\right)^a_i \gamma^5 \psi^b_j \right\rangle \sim \sum_{I,J=1}^{3} c^I_J \varepsilon_{ijI} \epsilon^{abJ} + \cdots, \tag{7}$$

which is antisymmetric in the color and flavor indices of the constituent quarks, cf. Eq. (4). The 3×3 matrix c^I_J is determined by the global minimum of the free energy. It appears that $c^I_J = \delta^I_J$. The ellipsis on the right hand side stand for a contribution which is symmetric in color and flavor. A small contribution of this type is always induced in the ground state, despite the fact that it corresponds to a repulsive diquark channel [18, 19]. This is not surprising after noting that the symmetric condensate, i.e., $\left\langle \left(\bar{\psi}^C\right)^a_i \gamma^5 \psi^b_j \right\rangle \sim \delta^a_i \delta^b_j + \delta^a_j \delta^b_i$, does not break any additional symmetries [18].

In the ground state, determined by the condensate (7), the chiral symmetry is broken down to its vector-like subgroup. The mechanism of this symmetry breaking is rather unusual, however. To see this clearly, it is helpful to rewrite the condensate as follows:

$$\left\langle \psi^{a,\alpha}_{L,i} \epsilon_{\alpha\beta} \psi^{b,\beta}_{L,j} \right\rangle = -\left\langle \psi^{a,\dot{\alpha}}_{R,i} \epsilon_{\dot{\alpha}\dot{\beta}} \psi^{b,\dot{\beta}}_{R,j} \right\rangle \sim \sum_{I=1}^{3} \varepsilon_{ijI} \epsilon^{abI} + \cdots, \tag{8}$$

where $\alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2$ are the spinor indices. The condensate of left-handed fields, $\left\langle \psi^{a,\alpha}_{L,i} \epsilon_{\alpha\beta} \psi^{b,\beta}_{L,j} \right\rangle$, breaks the $\text{SU}(3)_c$ color symmetry and the $\text{SU}(3)_L$ chiral symmetry, but leaves the diagonal $\text{SU}(3)_{L+c}$ subgroup unbroken. Indeed, as one can check, this condensate remains invariant under a flavor transformation (g_L) and a properly chosen compensating color transformation ($g_c = g_L^{-1}$). Similarly, the condensate of right-handed fields, $\left\langle \psi^{a,\dot{\alpha}}_{R,i} \epsilon_{\dot{\alpha}\dot{\beta}} \psi^{b,\dot{\beta}}_{R,j} \right\rangle$, leaves the $\text{SU}(3)_{R+c}$ subgroup unbroken.

When both condensates are present, the symmetry of the ground state is given by the diagonal subgroup $\text{SU}(3)_{L+R+c}$. This is because one has no freedom to use two different compensating color transformations. At the level of global symmetries, the original $\text{SU}(3)_L \times \text{SU}(3)_R$ symmetry of the model is broken down to the vector-like $\text{SU}(3)_{L+R}$, just like in vacuum. (Note, however, that the CFL phase is superfluid because the global $U(1)_B$ symmetry is broken by the diquark condensate in the ground state.) Unlike in vacuum, the chiral symmetry breaking does *not* result from any condensate mixing left- and right-handed fields. Instead, it results primarily from two separate condensates, made of left-handed fields and of right-handed fields only. The flavor orientations of the two condensates are "locked" to each

other by color transformations. This mechanism is called locking, and the corresponding phase of matter is called color-flavor-locked (CFL) phase [18].

The gap equation in the three-flavor quark matter is qualitatively the same as in the two-flavor case. The differences come only from a slightly more complicated color-flavor structure of the off-diagonal part of the inverse quark propagator (gap matrix) [18, 19],

$$\Delta_{ab}^{ij} = i\gamma^5 \left[\frac{1}{3} \left(\Delta_1 + \Delta_2 \right) \delta_a^i \delta_b^j - \Delta_2 \delta_b^i \delta_a^j \right], \tag{9}$$

where two parameters Δ_1 and Δ_2 determine the values of the gaps in the quasiparticles spectra. In the ground state, which is invariant under the $SU(3)_{L+R+c}$ symmetry, the original nine quark states give rise to a singlet and an octet of quasiparticles with different values of the gaps in their spectra. When a small color-symmetric diquark condensate is neglected, one finds that the gap of the singlet (Δ_1) is twice as large as the gap of the octet (Δ_2), i.e., $\Delta_1 = 2\Delta_2$. In general, however, this relation is only approximate. In QCD at asymptotic density, the dependence of the gaps on the quark chemical potential was calculated in Refs. [19, 20].

5 Dense matter inside stars

As discussed in Secs. 1 and 2, it is natural to expect that color superconducting phases may exist in the interior of compact stars. The estimated central densities of such stars might be sufficiently large for producing deconfined quark matter. Then, such matter develops the Cooper instability and becomes a color superconductor. It should also be noted that typical temperatures inside compact stars are so low that a spin-0 diquark condensate, if produced, would not melt. (Of course, this may not apply to a short period of the stellar evolution immediately after the supernova explosion.)

In the preceding sections, only idealized versions of dense matter, in which the Fermi momenta of pairing quarks were equal, were discussed. These cannot be directly applied to a realistic situation that is thought to occur inside compact stars. The reason is that matter in the bulk of a compact star should be neutral (at least, on average) with respect to electric as well as color charges. Also, matter should remain in β (chemical) equilibrium, i.e., the β processes $d \to u + e^- + \bar{\nu}_e$ and $u + e^- \to d + \nu_e$ (as well as $s \to u + e^- + \bar{\nu}_e$ and $u + e^- \to s + \nu_e$ in the presence of strange quarks) should go with equal rates. (Here it is assumed that there is no neutrino trapping in stellar matter. In the presence of neutrino trapping, the situation changes [21]. Also, the situation changes in the presence of a very strong magnetic field [22], but the discussion of its effect is outside the scope of this short review.)

Formally, β equilibrium is enforced by introducing a set of chemical potentials (μ_i) in the partition function of quark matter,

$$Z = \text{Tr} \exp \left(-\frac{H + \sum_i \mu_i Q_i}{T} \right). \tag{10}$$

The total number of independent chemical potentials μ_i is equal to the number of conserved charges Q_i in the model. For example, in two-flavor quark matter, it suffices to consider only three relevant conserved charges: the baryon number n_B, the electric charge n_Q, and the color charge n_8. (Note that these may not be sufficient in a general case [23].) Then, the matrix of quark chemical potentials is given in terms of the baryon chemical potential (by definition, $\mu_B \equiv 3\mu$), the electron chemical potential (μ_e) and the color chemical potential (μ_8) [24, 25, 26],

$$\hat{\mu}_{ij,\alpha\beta} = (\mu\delta_{ij} - \mu_e Q_{ij})\delta_{\alpha\beta} + \frac{2}{\sqrt{3}}\mu_8\delta_{ij}(T_8)_{\alpha\beta}, \tag{11}$$

where Q and T_8 are the generators of $U(1)_{em}$ of electromagnetism and the $U(1)_8$ subgroup of the gauge group $SU(3)_c$.

The other important condition in stellar matter is that of charge neutrality. In order to get an impression regarding the importance of charge neutrality in a large macroscopic chunk of matter, such

as a core of a compact star, one can estimate the corresponding Coulomb energy. A simple calculation leads to the following result:

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim 10^{26} M_\odot c^2 \left(\frac{n_Q}{10^{-2} e/\text{fm}^3} \right)^2 \left(\frac{R}{1\,\text{km}} \right)^5, \tag{12}$$

here R is the radius of the quark matter core, whose charge density is denoted by n_Q. It is easy to see that this energy is not an extensive quantity: the value of the corresponding *energy density* increases with the size of the system as R^2. By taking a typical value of the charge density in the 2SC phase, $n_Q \sim 10^{-2} e/\text{fm}^3$, the energy in Eq. (12) becomes a factor of 10^{26} larger than the rest mass energy of the Sun! To avoid such an incredibly large energy price, the charge neutrality $n_Q = 0$ should be satisfied with a very high precision.

In the case of two-flavor quark matter, one can argue that the neutrality is achieved if the number density of down quarks is approximately twice as large as number density of up quarks, $n_d \approx 2n_u$. This follows from the fact that the negative charge of the down quark ($Q_d = -1/3$) is twice as small as the positive charge of the up quark ($Q_u = 2/3$). When $n_d \approx 2n_u$, the total electric charge density is vanishing in absence of electrons, $n_Q \approx Q_d n_d + Q_u n_u \approx 0$. It turns out that even a nonzero density of electrons, required by the β equilibrium condition, could not change this relation much.

The argument goes as follows. One considers noninteracting massless quarks. In β equilibrium, the chemical potentials of the up quark and the down quark, μ_u and μ_d, should satisfy the relation $\mu_d = \mu_u + \mu_e$ where μ_e is the chemical potential of electrons (i.e., up to a sign, the chemical potential of the electric charge). By assuming that $\mu_d \approx 2^{1/3}\mu_u$, i.e., $n_d \approx 2n_u$ as required by the neutrality in absence of electrons, one obtains the following result for the electron chemical potential: $\mu_e = \mu_d - \mu_u \approx 0.26\mu_u$. The corresponding density of electrons is $n_e \approx 6 \cdot 10^{-3} n_u$, i.e., $n_e \ll n_u$ which is in agreement with the original assumption that $n_d \approx 2n_u$ in neutral matter.

While the approximate relation $n_d \approx 2n_u$ may be slightly modified in an interacting system, the main conclusion remains qualitatively the same. The Fermi momenta of up and down quarks, whose pairing is responsible for color superconductivity, are generally non-equal when neutrality and β equilibrium are imposed. This affects the dynamics of Cooper pairing and, as a consequence, some color superconducting phases may become less favored than others. For example, it is argued in Ref. [24], that a mixture of unpaired strange quarks and the non-strange 2SC phase, made of up and down quarks, is less favorable than the CFL phase after the charge neutrality condition is enforced. In addition, it was found that neutrality and β equilibrium may give rise to new unconventional pairing patterns [25, 27].

6 Different dynamical regimes in neutral matter

By studying neutral two-flavor quark matter, it was found that there exist three qualitatively different dynamical regimes, defined by the (largely unknown) strength of diquark coupling [25]. Similar regimes were also suggested to exist in three-flavor quark matter when the strange quark mass is not negligibly small [27, 28]. (Other effects due to a non-zero strange quark mass are discussed in Ref. [29].)

The simplest regime corresponds to weak diquark coupling. In this case, cross-flavor Cooper pairing of quarks with non-equal Fermi momenta is energetically unfavorable. The ground state of neutral matter corresponds to the normal phase. This would be precisely the case in QCD at asymptotic density if there existed only up and down quark flavors. (Formally, this is also the case when there are six quark flavors as in the Standard Model!) One should note, however, that a much weaker spin-1 pairing between quarks of same flavor is not forbidden in such neutral matter. In fact, spin-1 condensates would be inevitable if the temperature is sufficiently low.

The other limiting case is the strongly coupled regime. It is clear that, if the value of the diquark coupling is sufficiently large, the color condensation could be made as strong as needed to overcome a

finite mismatch between the Fermi surfaces of pairing quarks. In this regime, the ground state is in the 2SC/CFL phase because β-equilibrium and charge neutrality have little effect.

It turns out that there also exists an intermediate regime, in which the diquark coupling is neither too weak nor too strong. It was proposed that the ground state in this regime is given by the so-called gapless superconductor [25, 27], briefly discussed in the next section.

7 Gapless 2SC and CFL phases

Without going into details, the characteristic feature of a gapless superconducting phase is the existence of gapless quasiparticle excitations in its low-energy spectrum. The simplest examples are the gapless 2SC (g2SC) [25] and gapless CFL (gCFL) [27] phases. In the g2SC case, for example, there exists a doublet of quasiparticles with the following dispersion relation [25]:

$$E_\Delta(k) = \sqrt{(E_\mathbf{k} - \bar{\mu})^2 + \Delta^2} - \delta\mu, \tag{13}$$

where Δ is the value of the gap parameter, $\bar{\mu} \equiv (\mu_1 + \mu_2)/2$ is the average chemical potential and $\delta\mu \equiv (\mu_1 - \mu_2)/2$ is the mismatch between the chemical potentials of pairing quarks. When $\Delta < \delta\mu$, it takes vanishingly small amount of energy to excite quasiparticles with momenta in the vicinity of $k_\pm \equiv \bar{\mu} \pm \sqrt{(\delta\mu)^2 - \Delta^2}$. Similar quasiparticles also exist in gCFL phase as well.

When the g2SC and gCFL phases were suggested, it was argued that their thermodynamic stability was enforced by the charge neutrality condition [25]. In a homogeneous macroscopic system, such a condition is necessary in order to avoid a huge energy price due to the Coulomb long-range interaction. Remarkably, this condition has no analogue in solid state physics. Thus, one argued that the known problems of the so-called Sarma [30] phase may not apply to the g2SC/gCFL phases.

8 Chromomagnetic instability and suggested alternatives

Rather quickly, it was discovered that the gapless phases have problems of their own [31]. Namely, the screening Meissner masses of several gauge bosons are imaginary in the ground state, indicating a new type (chromomagnetic) instability in quark matter. The original calculation was performed for the g2SC phase [31], but a similar observation regarding the gCFL phase was also made soon [32, 33].

In the case of the g2SC phase, e.g., it was found that the screening Meissner masses for five out of total eight bosons are imaginary when $0 < \Delta/\delta\mu < 1$. In addition and most surprisingly, it was also found that four gauge bosons have imaginary masses even in the *gapped* 2SC phase when $1 < \Delta/\delta\mu < \sqrt{2}$. The most natural interpretation of these results is that the instability might be resolved through the formation of a gluon condensate in the ground state [31]. It is fair to note, however, that the exact nature of the instability (and in the case of $1 < \Delta/\delta\mu < \sqrt{2}$, in particular) is still poorly understood. The presence of the imaginary masses even in the *gapped* phase (i.e., when $1 < \Delta/\delta\mu < \sqrt{2}$), may suggest that the gapless superconductivity is not the only reason for the instability. While there remain many open questions, a partial progress in resolving the problem has already been made [34, 35].

In the gCFL phase, the instability is seen only for three gauge bosons [33]. The corresponding screening Meissner masses have a dependence on the mismatch parameter which is similar to the that for the 8th gluon in the g2SC phase. The fate of such an instability has not been clarified completely. At asymptotic density, however, it was suggested that the stable ground state might be given by a phase with an additional p-wave meson condensate [36]. Whether a similar phase also exists in two-flavor quark matter is unclear because the situation is further complicated there by (i) the absence of a natural mesonic state among the low-energy excitations and (ii) the onset of the "abnormal" chromomagnetic instability for the gluons A_μ^4, A_μ^5, A_μ^6, and A_μ^7. Instead of a p-wave meson condensate, the so-called "gluonic" phase may be realized [35].

The presence of the chromomagnetic instability in g2SC and gCFL phases indicates that these phases cannot be stable ground states of matter. It should be emphasized, however, that this does not

mean that, in nature, gapless phases are ruled out completely. First of all, there is an indication from studies in non-relativistic models that similar instabilities may not appear under some special conditions [37, 38]. In addition, most of the alternatives to the g2SC [25] and gCFL [27] phases, that have been suggested [34, 35, 36], share the same qualitative feature: their spectra of low-energy quasiparticles possess gapless modes. In fact, this seems to be not accidental but the most natural outcome of a very simple observation: the ordinary "gapped" versions of superconductivity are hardly consistent with the unconventional Cooper pairing, required in neutral and β-equilibrated quark matter.

9   Discussion

In conclusion, there has been a tremendous progress in recent studies of dense baryonic matter. This started from a seemingly innocuous observation that the size of the gap in the energy spectrum of color superconducting quark matter, under conditions realized in stars, could be of the same order as the QCD scale [9]. This opened a whole new chapter in studies of new states of dense matter that could exist inside compact stars. In addition to a phenomenological/observational interest, the recent studies in color superconductivity in neutral and β-equilibrated matter revealed a wide range of fundamentally new possibilities stemming from unconventional Cooper pairing. It is plausible that in the future a cross-disciplinary importance of this finding may even overshadow its role in physics of compact stars.

If color superconducting quark matter indeed exists in the interior of compact stars, it should affect some important transport and thermodynamic properties of stellar matter which may, in turn, affect some observational data from stars. Among the most promising signals are the cooling rates [39, 40] and the rotational slowing down of stars [41]. Also, new states of matter could affect the stellar mass-radius relation [42], and even lead to the existence of a new family of compact stars [43]. Color superconductivity can also affect directly as well as indirectly many other observed properties of stars. In some cases, for example, superconductivity may be accompanied by baryon superfluidity and/or the electromagnetic Meissner effect. If matter is superfluid, rotational vortices would be formed in the stellar core, and they would carry a portion of the angular momentum of the star. Because of the Meissner effect, the star interior could become threaded with magnetic flux tubes. In either case, the star evolution may be affected. While some studies on possible effects of color superconductivity in stars have already been attempted, the systematic study remains to be done in the future.

The development in the field also resulted in obtaining reliable nonperturbative solutions in QCD at asymptotic densities [10, 19, 20, 29], shedding some light on the structure of the QCD phase diagram in the regime inaccessible by lattice calculations. By itself, this has a fundamental theoretical importance. Also, this may provide valuable insights into the theory of strong interactions. One of the examples might be the idea of duality between the hadronic and quark description of QCD [44]. In the future, the structure of the QCD phase diagram and the properties of various color superconducting phases should be studied in more detail. While many different phases of quark matter have been proposed, there is no certainty that all possibilities have already been exhausted.

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