Spectral convexity for attractive $SU(2N)$ fermions

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Abstract

We prove a general theorem on spectral convexity with respect to particle number for $2N$ degenerate components of fermions. The number of spatial dimensions is arbitrary, and the system may be uniform or constrained by an external potential. We assume only that the interactions are governed by an $SU(2N)$-invariant two-body potential whose Fourier transform is negative definite. The convexity result implies that the ground state is in a $2N$-particle clustering phase. We discuss implications for light nuclei as well as asymmetric nuclear matter in neutron stars.

Interacting fermions with more than two components exhibit a variety of low temperature phenomena. Of particular interest are phenomena which appear in different quantum systems and therefore could be characterized as universal. One example in three dimensions is the Efimov effect, which predicts a geometric sequence of trimer bound states for interactions in the limit of zero range and infinite scattering length [1, 2, 3, 4, 5, 6]. The Efimov effect is forbidden for two-component fermions due to the Pauli exclusion principle but can occur for more than two components. Efimov trimers have recently been observed in ultracold cesium as indicated by a large three-body recombination loss near a Feshbach resonance [7]. Once the binding energy of the trimer system is fixed for interactions at zero range and large scattering length, the binding energy of the four-body system is also determined. This is in direct analogy with the Tjon line relating the nuclear binding energies of ^{3}H and ^{4}He [8, 9, 10]. In two dimensions a different geometric sequence has been predicted for zero-range attractive interactions. In this case the geometric sequence describes the binding energy of N-body clusters as a function of N in the large N limit [11, 12, 13, 14].

Several recent studies have investigated pairing and the superfluid properties of three-component fermions [15, 16, 17, 18]. Systems involving four-component fermions are of direct relevance to the low-energy effective theory of protons and neutrons. Due to antisymmetry there are only two S-wave nucleon scattering lengths, corresponding with the spin-singlet and spin-triplet channels. Some general properties of this low-energy effective theory have been studied such as pairing, the fermion sign problem, and spectral inequalities [19, 20, 21, 22]. Wu and collaborators [23, 24, 25] have pointed out that the effective theory has an accidental $SO(5)$ or $Sp(4)$ symmetry, and several different phases such as quintet Cooper pairing or four-fermion quartetting could be experimentally realized for different scattering lengths with ultracold atoms in optical traps or lattices [26, 27]. When the scattering lengths are equal the symmetry is expanded to $SU(4)$. This symmetry was first studied by Wigner [28] and arises naturally in the limit of large number of colors for quantum chromodynamics [29, 30]. The fact that both the spin-singlet and spin-triplet nucleon scattering lengths are unusually large means that the physics of low-energy nucleons is close to the Wigner limit [31, 32].

In the following we prove a general theorem on spectral convexity with respect to particle number for $2N$ degenerate components of fermions. The theorem holds for any number of spatial dimensions, and the system may be either uniform or constrained by an external

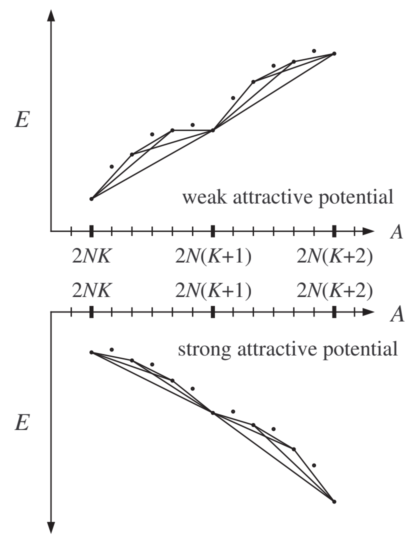

FIG. 1: Illustration of the convexity constraints for the ground state energy E as a function of particle number A. The line segments show the convexity lower bounds.

potential. We assume only that the interactions are governed by an $SU(2N)$-invariant two-body potential whose Fourier transform is negative definite. The main result is that if the ground state energy E is plotted as a function of the number of particles A, then the function $E(A)$ is convex for even A modulo $2N$. Furthermore $E(A)$ for odd A is bounded below by the average of the two neighboring even values, $E(A-1)$ and $E(A+1)$. This is illustrated in Fig. 1 for both the weak attractive and strong attractive cases. This convexity pattern could be regarded as an $SU(2N)$ generalization of even-odd staggering for the ground state energy in the attractive two-component system.

A weaker form of this inequality was proven for $A \leq 2N$ and zero-range attractive interactions [21]. Here we extend the proof to any A and any $SU(2N)$-invariant potential with a negative-definite Fourier transform. Another difference between this and the previous analysis is that we use a fixed particle number formalism rather than the grand canonical formalism. This is essential for strongly-attractive interactions and the requirement of keeping the physics in the low-energy regime. By limiting the number of particles we avoid a collapse towards high densities and the need for hard core repulsion for stability.

We start by considering $2N$ degenerate components of nonrelativistic fermions in d spatial dimensions. We assume the interactions are governed by an $SU(2N)$-invariant two-body potential $V(\vec{r})$ whose Fourier transform $\tilde{V}(\vec{p})$ is strictly negative. We also allow an $SU(2N)$-

invariant external potential $U(\vec{r})$ whose properties are not restricted. The general form of the Hamiltonian is

$$H = -\frac{1}{2m}\sum_{i=1,\cdots,2N}\int d^d\vec{r}\, a_i^\dagger(\vec{r})\vec{\nabla}^2 a_i(\vec{r}) + \int d^d\vec{r}\, U(\vec{r})\rho(\vec{r})$$
$$+\frac{1}{2}\int d^d\vec{r} d^d\vec{r}'\; :\rho(\vec{r})V(\vec{r}-\vec{r}')\rho(\vec{r}'): , \tag{1}$$

where $\rho(\vec{r})$ is the $SU(2N)$-invariant density,

$$\rho(\vec{r}) = \sum_{i=1,\cdots,2N} a_i^\dagger(\vec{r})a_i(\vec{r}). \tag{2}$$

The : symbols denote normal ordering. We consider the system on a hypercubic lattice using a transfer matrix formalism. We let $\vec{n} = (\vec{n}_s, n_t)$ represent $(d+1)$-dimensional lattice vectors. The subscript s on $\vec{n}_s$ denotes a d-dimensional spatial lattice vector. We write the d-dimensional spatial lattice unit vectors as $\hat{1},\cdots,\hat{d}$. Throughout our discussion of the lattice system we use dimensionless parameters and operators which correspond with physical values multiplied by the appropriate power of the spatial lattice spacing a. We let a_t be the temporal lattice spacing and α_t be the ratio a_t/a. L denotes the spatial length of the periodic hypercubic lattice.

We use the notation $\tilde{V}(2\pi\vec{k}_s/L)$ for the Fourier transform of the lattice potential $V(\vec{n}_s)$,

$$\tilde{V}(2\pi\vec{k}_s/L) = \sum_{\vec{n}_s} V(\vec{n}_s)e^{i2\pi\vec{n}_s\cdot\vec{k}_s/L}. \tag{3}$$

By assumption $\tilde{V}(2\pi\vec{k}_s/L)$ is strictly negative. Let M be the normal-ordered transfer matrix operator

$$M =:\exp\left[-\alpha_t H_{\text{free}} - \alpha_t\sum_{\vec{n}_s}U(\vec{n}_s)\rho(\vec{n}_s) - \frac{\alpha_t}{2}\sum_{\vec{n}_s,\vec{n}_s'}\rho(\vec{n}_s)V(\vec{n}_s-\vec{n}_s')\rho(\vec{n}_s')\right]: , \tag{4}$$

where H_{free} is the free lattice Hamiltonian,

$$H_{\text{free}} = -\frac{1}{2m}\sum_{\vec{n}_s}\sum_{\hat{l}_s=\hat{1},\cdots,\hat{d}}\sum_{i=1,\cdots,2N}\left\{a_i^\dagger(\vec{n}_s)\left[a_i(\vec{n}_s+\hat{l}_s)+a_i(\vec{n}_s-\hat{l}_s)-2a_i(\vec{n}_s)\right]\right\}. \tag{5}$$

Let $V^{-1}(\vec{n}_s)$ be the inverse of $V(\vec{n}_s)$,

$$V^{-1}(\vec{n}_s) = \frac{1}{L^d}\sum_{\vec{k}_s}\frac{e^{-i2\pi\vec{n}_s\cdot\vec{k}_s/L}}{\tilde{V}(2\pi\vec{k}_s/L)}. \tag{6}$$

We now rewrite powers of M using an auxiliary field ϕ,

$$M^{L_t} = \int D\phi \; e^{-S(\phi)} \; M_{L_t-1}(\phi) \times \cdots \times M_0(\phi), \tag{7}$$

where

$$S(\phi) = -\frac{\alpha_t}{2} \sum_{n_t} \sum_{\vec{n}_s, \vec{n}'_s} \phi(\vec{n}_s, n_t) V^{-1}(\vec{n}_s - \vec{n}'_s) \phi(\vec{n}'_s, n_t), \tag{8}$$

$$M_{n_t}(\phi) \equiv : \exp\left[-\alpha_t H_{\text{free}} - \alpha_t \sum_{\vec{n}_s} U(\vec{n}_s)\rho(\vec{n}_s) + \alpha_t \sum_{\vec{n}_s} \phi(\vec{n}_s, n_t)\rho(\vec{n}_s)\right] : , \tag{9}$$

$$D\phi = \prod_{\vec{k}_s} \left[-\tilde{V}(2\pi\vec{k}_s/L)\right]^{-L_t/2} \times \prod_{\vec{n}_s, n_t} \frac{d\phi(\vec{n}_s, n_t)}{\sqrt{2\pi/\alpha_t}}. \tag{10}$$

Let $f^{(1)}(\vec{n}_s), f^{(2)}(\vec{n}_s), \cdots$ be a complete set of orthonormal real-valued functions of the spatial lattice sites $\vec{n}_s$. We refer to these functions as orbitals. We take $f^{(1)}(\vec{n}_s)$ to be strictly positive but otherwise regard the form for the orbitals to be arbitrary. If the total number of lattice sites is L^d then we have a total of L^d orbitals. We denote a one-particle state with component i in the k^{th} orbital as $\left| f_i^{(k)} \right\rangle$.

Let $\mathcal{B}$ and $\mathcal{C}$ be any finite subsets of the orbital indices, $\mathcal{B}, \mathcal{C} \subset \{1, 2, \cdots, L^d\}$. From these we define $\left|\mathcal{B}^j \mathcal{C}^{2N-j}\right\rangle$ as the quantum state where each of j components fill orbitals $\mathcal{B}$ and each of the remaining $2N-j$ components fill the orbitals $\mathcal{C}$. The order of the component labels is irrelevant, and so we assume that the first j components fill orbitals $\mathcal{B}$ and last $2N-j$ components fill orbitals $\mathcal{C}$. The total number of fermions in state $\left|\mathcal{B}^j \mathcal{C}^{2N-j}\right\rangle$ is $j\,|\mathcal{B}| + (2N-j)\,|\mathcal{C}|$, where $|\mathcal{B}|$ and $|\mathcal{C}|$ are the number of elements in $\mathcal{B}$ and $\mathcal{C}$ respectively.

We define $E_{\mathcal{B}^j\mathcal{C}^{2N-j}}$ as the energy of the lowest energy eigenstate with nonzero inner product with $\left|\mathcal{B}^j \mathcal{C}^{2N-j}\right\rangle$. We let $Z^{L_t}_{\mathcal{B}^j\mathcal{C}^{2N-j}}$ be the expectation value of M^{L_t} for $\left|\mathcal{B}^j \mathcal{C}^{2N-j}\right\rangle$,

$$Z^{L_t}_{\mathcal{B}^j\mathcal{C}^{2N-j}} = \left\langle \mathcal{B}^j \mathcal{C}^{2N-j} \right| M^{L_t} \left| \mathcal{B}^j \mathcal{C}^{2N-j} \right\rangle. \tag{11}$$

In the limit of large L_t the contribution from the lowest energy eigenstate dominates and therefore

$$E_{\mathcal{B}^j\mathcal{C}^{2N-j}} = -\lim_{L_t\to\infty} \frac{\ln\left(Z^{L_t}_{\mathcal{B}^j\mathcal{C}^{2N-j}}\right)}{\alpha_t L_t}. \tag{12}$$

We can write $Z^{L_t}_{\mathcal{B}^j\mathcal{C}^{2N-j}}$ using the auxiliary field ϕ,

$$Z^{L_t}_{\mathcal{B}^j\mathcal{C}^{2N-j}} = \int D\phi \; e^{-S(\phi)} \; \left\langle \mathcal{B}^j \mathcal{C}^{2N-j} \right| M_{L_t-1}(\phi) \times \cdots \times M_0(\phi) \left| \mathcal{B}^j \mathcal{C}^{2N-j} \right\rangle. \tag{13}$$

At this point we define matrix elements for the one particle states,

$$\mathcal{M}_{k',k}(\phi) = \left\langle f_i^{(k')} \right| M_{L_t-1}(\phi) \times \cdots \times M_0(\phi) \left| f_i^{(k)} \right\rangle . \tag{14}$$

The component index i in (14) does not matter due to the $SU(2N)$ symmetry. Each entry of the matrix $\mathcal{M}_{k',k}(\phi)$ is real. We let $\mathcal{M}_{\mathcal{B}}(\phi)$ be the $|\mathcal{B}| \times |\mathcal{B}|$ submatrix consisting of the rows and columns in $\mathcal{B}$ and let $\mathcal{M}_{\mathcal{C}}(\phi)$ be the $|\mathcal{C}| \times |\mathcal{C}|$ submatrix for $\mathcal{C}$.

Each normal-ordered transfer matrix operator $M_{n_t}(\phi)$ has only single-particle interactions with the auxiliary field and no direct interactions between particles. Therefore it follows that

$$Z^{L_t}_{\mathcal{B}^j\mathcal{C}^{2N-j}} = \int D\phi \; e^{-S(\phi)} \; \left[\det \mathcal{M}_{\mathcal{B}}(\phi)\right]^j \left[\det \mathcal{M}_{\mathcal{C}}(\phi)\right]^{2N-j} . \tag{15}$$

This result is perhaps more transparent if we pretend for the moment that each of the $j\,|\mathcal{B}| + (2N-j)\,|\mathcal{C}|$ particles carries an extra quantum number which makes them distinguishable. We label the extra quantum number as X. So long as the initial and final state wavefunctions are completely antisymmetric in X for particles of the same component then this error in quantum statistics has no effect on the final amplitude. So we can factorize the transfer matrices $M_{n_t}(\phi)$ as a product of transfer matrices for each X. This then leads directly to (15).

Let n_1 and n_2 be integers such that $0 \leq 2n_1 < j < 2n_2 \leq 2N$. Let us define the new positive-definite measure

$$\tilde{D}\phi = D\phi \; e^{-S(\phi)} \; \left[\det \mathcal{M}_{\mathcal{B}}(\phi)\right]^{2n_1} \left[\det \mathcal{M}_{\mathcal{C}}(\phi)\right]^{2N-2n_2} , \tag{16}$$

so that

$$Z^{L_t}_{\mathcal{B}^j\mathcal{C}^{2N-j}} = \int \tilde{D}\phi \; \left[\det \mathcal{M}_{\mathcal{B}}(\phi)\right]^{j-2n_1} \left[\det \mathcal{M}_{\mathcal{C}}(\phi)\right]^{2n_2-j} . \tag{17}$$

Then by the Hölder inequality $\left|Z^{L_t}_{\mathcal{B}^j\mathcal{C}^{2N-j}}\right|$ is bounded above by

$$\left[\int \tilde{D}\phi \; \left|\det \mathcal{M}_{\mathcal{B}}(\phi)\right|^{2n_2-2n_1}\right]^{\frac{j-2n_1}{2n_2-2n_1}} \left[\int \tilde{D}\phi \; \left|\det \mathcal{M}_{\mathcal{C}}(\phi)\right|^{2n_2-2n_1}\right]^{\frac{2n_2-j}{2n_2-2n_1}}$$
$$= \left(Z^{L_t}_{\mathcal{B}^{2n_2}\mathcal{C}^{2N-2n_2}}\right)^{\frac{j-2n_1}{2n_2-2n_1}} \left(Z^{L_t}_{\mathcal{B}^{2n_1}\mathcal{C}^{2N-2n_1}}\right)^{\frac{2n_2-j}{2n_2-2n_1}} . \tag{18}$$

Taking the limit $L_t \to \infty$ we deduce that the energies satisfy the inequality

$$E_{\mathcal{B}^j\mathcal{C}^{2N-j}} \geq \tfrac{j-2n_1}{2n_2-2n_1} E_{\mathcal{B}^{2n_2}\mathcal{C}^{2N-2n_2}} + \tfrac{2n_2-j}{2n_2-2n_1} E_{\mathcal{B}^{2n_1}\mathcal{C}^{2N-2n_1}} . \tag{19}$$

This is a statement of convexity for $E_{\mathcal{B}^j\mathcal{C}^{2N-j}}$ as a function of j between even endpoints $j = 2n_1$ and $j = 2n_2$. If we now take $|\mathcal{B}| = K + 1$ and $|\mathcal{C}| = K$, then the total particle number is $A = 2NK + j$ and A lies between $2NK$ and $2N(K + 1)$. The inequality in (19) is precisely the convexity pattern in Fig. 1 for $E(A)$ as a function of particle number.

We point out that for the special case $K = 0$, we can take $\mathcal{B}$ to be the first orbital and $\mathcal{C}$ to be the empty set. In this case $\mathcal{M}_{\mathcal{B}}(\phi)$ is simply a number. Furthermore since $f^{(1)}(\vec{n}_s)$ is strictly positive, $\mathcal{M}_{\mathcal{B}}(\phi)$ is also positive so long as the temporal lattice step a_t is not excessively large. Since $\det \mathcal{M}_{\mathcal{B}}(\phi) = \mathcal{M}_{\mathcal{B}}(\phi) > 0$ it is no longer necessary that the power of $\det \mathcal{M}_{\mathcal{B}}(\phi)$ be even to insure positivity. Therefore $E(A)$ is actually convex for all A between 0 and $2N$ and not just even A.

These convexity relations could be checked using any number of attractive $SU(2N)$ models in various dimensions. This will be checked in future studies. Here we instead examine actual nuclear physics data to investigate Wigner's approximate $SU(4)$ symmetry in light nuclei. It is by no means clear that the interactions of nucleons in light nuclei can be approximately described by an attractive $SU(4)$-symmetric potential. Recent results from nuclear lattice simulations hint that this might be possible [33, 34], however there are forces even at lowest order in chiral effective field theory which break $SU(4)$ invariance in addition to being repulsive. Nevertheless all of the $SU(4)$ convexity constraints are in fact satisfied for the most stable light nuclei with up to 16 nucleons as can be seen in Fig. 2. The line segments drawn show all of the convexity lower bounds.

There have been several recent studies of alpha clustering in nuclear matter [35, 36, 37, 38, 39, 40] as well as multiparticle clustering in other systems [24, 25, 26, 27, 41, 42]. The results presented here give sufficient conditions for the onset of this multiparticle clustering phase. One can also make a definite prediction about the j-component quasiparticle energy gaps. Starting from a $2NK$-fermion $SU(2N)$-symmetric state, let δ_j be the extra energy required per fermion to add j fermions, all of different components. The ground state energy for $2NK + j$ fermions is a convex function for even j in the interval from $j = 0$ to $j = 2N$. Therefore it follows that $\delta_2 \geq \delta_4 \geq \cdots \geq \delta_{2N}$. Since the ground state energy for $2NK + j$ fermions is also convex for $j = 0, 1, 2$, we conclude furthermore that $\delta_1 \geq \delta_2 \geq \delta_4 \cdots \geq \delta_{2N}$. We note that for the strongly attractive case these energy gaps are negative, and it is more natural to speak of energy gaps per missing fermion for the corresponding j-component quasiholes, δ_j^h. In this case we find again $\delta_1^h \geq \delta_2^h \geq \delta_4^h \cdots \geq \delta_{2N}^h$.

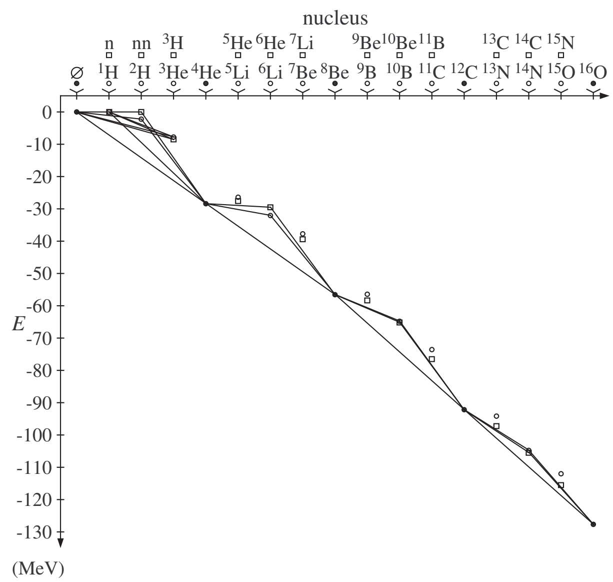

FIG. 2: Plot of the energy versus particle number for the most stable light nuclei with up to 16 nucleons. The line segments show the convexity lower bounds.

In summary we have derived a general result on spectral convexity with respect to particle number for $2N$ degenerate components of fermions. We assume only that the interactions are governed by an $SU(2N)$-invariant two-body potential whose Fourier transform is negative definite. The ground state energy E as a function of the number of particles A is convex for even A modulo $2N$. Also $E(A)$ for odd A is bounded below by the average of the two neighboring even values, $E(A-1)$ and $E(A+1)$. When applied to light nuclei for $A \leq 16$ all of the convexity bounds for $SU(4)$ are satisfied. These results give further evidence that an approximate description of light nuclei may be possible using an attractive $SU(4)$-symmetric potential. This would be a direction worth pursuing since the same theory could then be applied to dilute neutron-rich matter with a finite number of protons. The residual $SU(2) \times SU(2)$ symmetry for proton spins and neutron spins would guarantee that the Monte Carlo simulation could be done without fermion sign oscillations. The physics of this quantum system would be helpful in understanding the superfluid properties of dilute neutron-rich matter in the inner crust of neutron stars.

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[1] V. N. Efimov, Sov. J. Nucl. Phys. **12**, 589 (1971).

[2] V. N. Efimov, Phys. Rev. **C47**, 1876 (1993).

[3] P. F. Bedaque, H.-W. Hammer, and U. van Kolck, Phys. Rev. Lett. **82**, 463 (1999), nucl-th/9809025.

[4] P. F. Bedaque, H.-W. Hammer, and U. van Kolck, Nucl. Phys. **A646**, 444 (1999), nucl-th/9811046.

[5] P. F. Bedaque, H.-W. Hammer, and U. van Kolck, Nucl. Phys. **A676**, 357 (2000), nucl-th/9906032.

[6] E. Braaten and H.-W. Hammer, Phys. Rept. **428**, 259 (2006).

[7] T. Kraemer, M. Mark, P. Waldburger, J. G. Danzl, C. Chin, B. Engeser, A. D. Lange, K. Pilch, A. Jaakkola, H.-C. Naegerl, et al., Nature **440**, 315 (2006), cond-mat/0512394.

[8] L. Platter, H.-W. Hammer, and U.-G. Meißner, Phys. Rev. **A70**, 052101 (2004), cond-mat/0404313.

[9] L. Platter, H.-W. Hammer, and U.-G. Meißner, Phys. Lett. **B607**, 254 (2005), nucl-th/0409040.

[10] H. W. Hammer and L. Platter (2006), nucl-th/0610105.

[11] H.-W. Hammer and D. T. Son, Phys. Rev. Lett. **93**, 250408 (2004), cond-mat/0405206.

[12] L. Platter, H.-W. Hammer, and U.-G. Meißner, Few Body Syst. **35**, 169 (2004).

[13] D. Blume, Phys. Rev. **B72**, 094510 (2005), cond-mat/0507729.

[14] D. Lee, Phys. Rev. **A73**, 063204 (2006), physics/0512085.

[15] P. F. Bedaque and J. P. D'Incao, cond-mat/0602525.

[16] L. He, M. Jin, and P. Zhuang, Phys. Rev. **A74**, 033604 (2006), cond-mat/0604580.

[17] H. Zhai, cond-mat/0607459.

[18] T. Paananen, J.-P. Martikainen, and P. Torma, Phys. Rev. **A73**, 053606 (2006), cond-mat/0603498.

[19] D. Lee, Phys. Rev. **C70**, 064002 (2004), nucl-th/0407088.

[20] D. Lee, Phys. Rev. **C71**, 044001 (2005), nucl-th/0407101.

[21] J.-W. Chen, D. Lee, and T. Schaefer, Phys. Rev. Lett. **93**, 242302 (2004), nucl-th/0408043.

[22] C. Wu and S.-C. Zhang, Phys. Rev. **B71**, 155115 (2005), cond-mat/0407272.

[23] C. Wu, J.-p. Hu, and S.-c. Zhang, Phys. Rev. Lett. **91**, 186402 (2003).

[24] C. Wu, Phys. Rev. Lett. **95**, 266404 (2005).

[25] C. Wu, Mod. Phys. Lett. **B20**, 1707 (2006), cond-mat/0608690.

[26] P. Lecheminant, E. Boulat, and P. Azaria, Phys. Rev. Lett. **95**, 240402 (2005), cond-mat/0505468.

[27] S. Capponi, G. Roux, P. Azaria, E. Boulat, and P. Lecheminant, cond-mat/0612456.

[28] E. Wigner, Phys. Rev. **51**, 106 (1939).

[29] D. B. Kaplan and M. J. Savage, Phys. Lett. **B365**, 244 (1996), hep-ph/9509371.

[30] D. B. Kaplan and A. V. Manohar, Phys. Rev. **C56**, 76 (1997), nucl-th/9612021.

[31] T. Mehen, I. W. Stewart, and M. B. Wise, Phys. Rev. Lett. **83**, 931 (1999), hep-ph/9902370.

[32] E. Epelbaum, U. G. Meißner, W. Gloeckle, and C. Elster, Phys. Rev. **C65**, 044001 (2002), nucl-th/0106007.

[33] B. Borasoy, H. Krebs, D. Lee, and U.-G. Meißner, Nucl. Phys. **A768**, 179 (2006), nucl-th/0510047.

[34] B. Borasoy, E. Epelbaum, H. Krebs, D. Lee, and U.-G. Meissner (2006), nucl-th/0611087.

[35] G. Ropke, A. Schnell, P. Schuck, and P. Nozieres, Phys. Rev. Lett. **80**, 3177 (1998), cond-mat/9803325.

[36] A. Tohsaki, H. Horiuchi, P. Schuck, and G. Ropke, Phys. Rev. Lett. **87**, 192501 (2001), nucl-th/0110014.

[37] Y. Funaki, H. Horiuchi, A. Tohsaki, P. Schuck, and G. Ropke, Prog. Theor. Phys. **108**, 297 (2002), nucl-th/0207022.

[38] P. Schuck, H. Horiuchi, G. Ropke, and A. Tohsaki (2003), nucl-th/0302076.

[39] T. Yamada and P. Schuck, Phys. Rev. **C69**, 024309 (2004), nucl-th/0310077.

[40] A. Sedrakian, H. Muther, and P. Schuck (2004), nucl-th/0407020.

[41] P. Schlottmann, J. Phys.: Condens. Matter **6**, 1359 (1994).

[42] H. Kamei and K. Miyake, J. Phys. Soc. Jpn. **74**, 1911 (2005), cond-mat/0505468.