

Unstable particles as open quantum systems

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We present the probability-preserving description of the decaying particle within the framework of quantum mechanics of open systems, taking into account the superselection rule prohibiting the superposition of the particle and vacuum. In our approach the evolution of the system is given by a family of completely positive trace-preserving maps forming a one-parameter dynamical semigroup. We give the Kraus representation for the general evolution of such systems, which allows one to write the evolution for systems with two or more particles. Moreover, we show that the decay of the particle can be regarded as a Markov process by finding explicitly the master equation in the Lindblad form. We also show that there are remarkable restrictions on the possible strength of decoherence.

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I. INTRODUCTION

Recently, the tests of Bell inequalities [1] in the system of correlated neutral kaons [2, 3] or B mesons [4] has attracted some attention. The crucial point in studying correlations in this system are the oscillations of the strangeness and bottom, respectively. However, the instability of kaons makes the analysis of correlation experiments difficult. The state of the complete system is a superposition (or a mixture) of the states of the decaying particles and the decay products. The whole system undergoes a unitary evolution described usually in terms of quantum field theory. On the other hand, in correlation experiments of Einstein–Podolsky–Rosen–Bohm type [5, 6], it is more useful to neglect the evolution of decay products and consider solely the decaying particles. Unfortunately, such a description within the framework of quantum mechanics referring to the case with finite degrees of freedom leads to some difficulties. This is usually done by means of the Weisskopf–Wigner approach [7, 8], where the probability of detecting the particle is not conserved during the time evolution and, therefore, the Hamiltonian in such theories must be non-Hermitian. Moreover, this formalism leads to some ambiguities when applied to the description of correlation experiments. The reason is the probability loss caused by the decrease of the trace of the reduced density operator. This prevents one to calculate unambiguously the probability of finding the system in a given state after the projective measurement. Therefore, in our opinion, we need an approach enabling a description of the system that can be in two-particle states as well as the one-particle and even zero-particle states, which can arise during the time evolution (decay) of the initial system. It seems to us that the mentioned issues can be resolved by an assumption that the decaying particle can be found in a particle state as well as in the state of *the absence of the particle*, i.e., in the *vacuum* state

(it is not a vacuum in the sense used in quantum field theory, but rather in a sense used in [9]).

In this paper, we give the probability preserving description of the decaying particle within the framework of quantum mechanics of open systems. This approach is introduced in Sec. II, where the evolution of the system is given by a family of completely positive trace preserving maps forming a one-parameter dynamical semigroup [10, 11, 12]. Thus, in our approach the Hamiltonian is Hermitian and therefore the reduced density operator has a unit trace. We also find the operator-sum representation (Kraus representation) for the evolution of such systems, which immediately allows one to write the evolution for systems of two or more noninteracting particles. This is useful if we study quantum correlations between unstable particles. We would like to point out that we use the dynamical semigroup approach for the entire evolution of the unstable particle, not only for the description of its decoherence, as was done in [3, 13, 14, 15, 16]. Finally, in Sec. III we study the restrictions on the possible strength of decoherence that arise as a side effect of completely positive evolution of the system as well as we estimate the upper bound for the decoherence strength for K^0 and B^0 mesons.

II. THE TIME EVOLUTION OF UNSTABLE PARTICLES

In this section we discuss the evolution of unstable particles, neglecting their spatial degrees of freedom. In order to sketch our approach, we begin with the discussion of the case of the neutral pion. This example is rather elementary, but it helps us to illustrate the main idea of our approach. Next, we go to K^0 (B^0) mesons. The evolution of these particles is more complicated because of the phenomenon of transmutation between K^0 and \bar{K}^0 (B^0 and \bar{B}^0). We shall regard them as open systems, it means that their evolution is not unitary, but it must be treated as a one-parameter family of quantum operations forming a dynamical semigroup. Consequently, the density operator of the system must obey the master equation rather than the von Neumann equation.

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A. Unstable (pseudo)scalar particle

In this section we describe briefly the evolution of an unstable (pseudo)scalar particle, π^0 , and we show that this evolution can be regarded as a family of amplitude damping quantum operations [17].

The key point of the presented approach is that the system under consideration can be regarded as a two-level system: one can find the system in the particle state or in the vacuum state. Of course, this system must be an open one and we treat the decay products as a part of the environment.

The space of states of the system is a direct sum of the Hilbert space of the particle \mathcal{H}_{π^0} , spanned by the vector $|\pi^0\rangle$, and the Hilbert space of the vacuum \mathcal{H}_0 , spanned by the vector $|0\rangle$; i.e., $\mathcal{H} = \mathcal{H}_{\pi^0} \oplus \mathcal{H}_0$. Since we are looking for a description of a decaying particle in terms of quantum mechanics with finite degrees of freedom (not in the language of field theory; cf. [18]) we assume that the decay process is a Markov process and it is not described in the dynamical manner (i.e., it is not governed by a Hamiltonian).

We represent the vectors $|\pi^0\rangle$ and $|0\rangle$ by

$$|\pi^0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (1)$$

The time evolution of the system can be represented by the continuous one-parameter family of linear superoperators \mathcal{S}_t such that

$$\hat{\rho}(t) = \mathcal{S}_t \hat{\rho}(0), \quad (2)$$

where $\hat{\rho}(t)$ is the density operator of the system at the time t . These superoperators must be trace-preserving completely positive maps and they must form a one-parameter semigroup (see, e.g., [19, 20] and references therein), i.e.,

$$\text{tr}[\mathcal{S}_t \hat{\rho}(0)] = \text{tr}[\hat{\rho}(0)] = 1, \quad (3a)$$

$$\mathcal{S}_{t_1+t_2} = \mathcal{S}_{t_1} \mathcal{S}_{t_2}, \quad \forall t_1, t_2 \geq 0, \quad (3b)$$

and the map $t \mapsto \mathcal{S}_t$ is continuous in strong topology.

We study the time evolution of the state of the system,

$$\hat{\rho} = \rho_{11} |\pi^0\rangle \langle \pi^0| + \rho_{12} |\pi^0\rangle \langle 0| + \rho_{12}^* |0\rangle \langle \pi^0| + \rho_{22} |0\rangle \langle 0|, \quad (4)$$

assuming that it is consistent with phenomenological Weisskopf–Wigner evolution [7, 8],

$$|\pi^0(t)\rangle = e^{-t(im+\Gamma/2)} |\pi^0\rangle, \quad (5)$$

where m is the π^0 mass and Γ is its decay width.

From (5), it follows that $\rho_{11}(t) = e^{-t\Gamma} \rho_{11}(0)$ and therefore $\rho_{22}(t) = 1 - e^{-t\Gamma} \rho_{11}(0)$. Taking into account the linearity of \mathcal{S}_t , we can write $\rho_{12}(t)$ as the time-dependent linear combinations of all the elements of the initial density matrix, i.e. $\rho_{12}(t) = \sum_{i,j=1}^2 A_{ij}(t) \rho_{ij}(0)$, with the initial conditions $A_{12}(0) = 1$ and all remaining A 's vanish at $t = 0$.

Therefore the action of the map \mathcal{S}_t can be written as follows:

$$\mathcal{S}_t \rho(0) = \rho(t) = \begin{pmatrix} e^{-t\Gamma} \rho_{11}(0) & \sum_{i,j=1}^2 A_{ij}(t) \rho_{ij}(0) \\ \sum_{i,j=1}^2 A_{ij}^*(t) \rho_{ij}^*(0) & \rho_{22}(t) \end{pmatrix}, \quad (6)$$

where

$$\rho_{22}(t) = \rho_{22}(0) + (1 - e^{-t\Gamma}) \rho_{11}(0). \quad (6a)$$

To find conditions under which the map (6) is completely positive, we use the Choi's theorem [19, 21], which states that \mathcal{S}_t is completely positive iff the corresponding Choi's matrix,

$$\text{Choi } \mathcal{S}_t = \begin{pmatrix} e^{-t\Gamma} & A_{11}(t) & 0 & A_{12}(t) \\ A_{11}^*(t) & 1 - e^{-t\Gamma} & A_{21}^*(t) & 0 \\ 0 & A_{21}(t) & 0 & A_{22}(t) \\ A_{12}^*(t) & 0 & A_{22}^*(t) & 1 \end{pmatrix} \quad (7)$$

is positive. This implies that

$$|A_{12}(t)|^2 \leq e^{-t\Gamma}, \quad (8a)$$

$$|A_{11}(t)|^2 \leq (1 - e^{-t\Gamma}) (e^{-t\Gamma} - |A_{12}(t)|^2), \quad (8b)$$

$$A_{21}(t) = 0, \quad (8c)$$

$$A_{22}(t) = 0. \quad (8d)$$

One can check by straightforward calculation that the composition law (3b) leads to the conditions

$$A_{12}(t_1 + t_2) = A_{12}(t_2) A_{12}(t_1), \quad (9a)$$

$$A_{11}(t_1 + t_2) = A_{11}(t_2) e^{-t_1\Gamma} + A_{12}(t_2) A_{11}(t_1). \quad (9b)$$

It is easy to see that the only possible solution of (9a) fulfilling (8a) and the initial conditions is

$$A_{12}(t) = e^{-t[(\Gamma+\lambda)/2+i\mu]}, \quad (10)$$

where $\lambda \geq 0$ and $\mu \geq 0$. Taking into account the fact that the one-parameter semigroup must be Abelian, we get from (9b), and (10)

$$A_{11}(t) = \begin{cases} z \left\{ e^{-t\Gamma} - e^{-t[(\Gamma+\lambda)/2+i\mu]} \right\}, & \text{when } \lambda \neq \Gamma \text{ or } \mu \neq 0, \\ z t e^{-t\Gamma}, & \text{when } \lambda = \Gamma \text{ and } \mu = 0, \end{cases} \quad (11)$$

where $z \in \mathbb{C}$ are such that the inequality (8b) is satisfied (for $\lambda = 0$ we have to put $z = 0$). Therefore, the most general form of the time-dependent density matrix is given by

$$\rho(t) = \begin{pmatrix} e^{-t\Gamma} \rho_{11}(0) & e^{-t[(\Gamma+\lambda)/2+im]} \rho_{12}(0) + A_{11}(t) \rho_{11}(0) \\ e^{-t[(\Gamma+\lambda)/2-im]} \rho_{12}^*(0) + A_{11}^*(t) \rho_{11}(0) & 1 - e^{-t\Gamma} \rho_{11}(0) \end{pmatrix}, \quad (12)$$

where the consistency with (5) requires $\mu = m$. The parameter λ is interpreted as the decoherence parameter.

Since the evolution of the system (12) is given by a completely positive map, it can also be written in the operator-sum form [22]

$$\hat{\rho}(t) = \sum_{i=0}^N \hat{E}_i(t) \hat{\rho}(0) \hat{E}_i^\dagger(t), \quad (13)$$

where the Kraus operators $\hat{E}_i(t)$ satisfy the condition $\sum_{i=0}^N \hat{E}_i^\dagger(t) \hat{E}_i(t) = I$. One can easily check that the Kraus operators leading to the evolution (12) are given by

$$\hat{E}_0(t) = e^{-t[(\Gamma+\lambda)/2+im]} |\pi^0\rangle \langle \pi^0| + |0\rangle \langle 0|, \quad (14a)$$

$$\hat{E}_1(t) = \sqrt{1 - e^{-t\Gamma} - \frac{|A_{11}(t)|^2 e^{t\Gamma}}{1 - e^{-t\lambda}}} |0\rangle \langle \pi^0|, \quad (14b)$$

$$\hat{E}_2(t) = e^{-t\Gamma/2} \sqrt{1 - e^{-t\lambda}} |\pi^0\rangle \langle \pi^0| + \frac{A_{11}^*(t) e^{t\Gamma/2}}{\sqrt{1 - e^{-t\lambda}}} |0\rangle \langle \pi^0|. \quad (14c)$$

From the operator-sum representation, using standard procedures [23, 24], we can easily find the local form of the time evolution—the master equation in the Lindblad form [12],

$$\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}, \hat{\rho}(t)] + \{\hat{K}, \hat{\rho}(t)\} + \sum_{i=1}^N \hat{L}_i \hat{\rho}(t) \hat{L}_i^\dagger, \quad (15)$$

where \hat{H} is the Hamiltonian of the system, the operators \hat{L}_i are the Lindblad operators, and $\hat{K} = -\frac{1}{2} \sum_{i=1}^N \hat{L}_i^\dagger \hat{L}_i$. For the density operator (12), the Hamiltonian is

$$\hat{H} = m |\pi^0\rangle \langle \pi^0|, \quad (16a)$$

and the Lindblad operators are of the form

$$\hat{L}_1 = \sqrt{\Gamma(1-\alpha)} |0\rangle \langle \pi^0|, \quad (16b)$$

$$\hat{L}_2 = \sqrt{\lambda} |\pi^0\rangle \langle \pi^0| + \frac{\beta^*}{\sqrt{\lambda}} |0\rangle \langle \pi^0|, \quad (16c)$$

where

$$\alpha = \begin{cases} |z|^2 [4m^2 + (\Gamma - \lambda)^2] / (4\Gamma\lambda), & \lambda \neq \Gamma \text{ or } m \neq 0, \\ |z|^2 / \Gamma^2, & \lambda = \Gamma \text{ and } m = 0, \end{cases} \quad (17a)$$

$$\beta = \begin{cases} z[im + (\Gamma - \lambda)/2], & \lambda \neq \Gamma \text{ or } m \neq 0, \\ z, & \lambda = \Gamma \text{ and } m = 0. \end{cases} \quad (17b)$$

Now, we take into account the fact that superpositions of the particle and vacuum are not observed in the nature. Consequently, there is no physical observable with nonvanishing matrix elements between vacuum state and particle state,

which leads to the *superselection rule*. Therefore, the element ρ_{12} of the density operator (4) does not contribute to the expectation value of any observable, and we can assume that $\rho_{12}(t) = 0$ for any time [25], which implies that $z = 0$. Therefore in this case the decoherence parameter λ becomes irrelevant, so we are free to put $\lambda = 0$. Thus the density matrix describing the evolution of π^0 is

$$\rho(t) = \begin{pmatrix} e^{-t\Gamma} \rho_{11}(0) & 0 \\ 0 & 1 - e^{-t\Gamma} \rho_{11}(0) \end{pmatrix}. \quad (18)$$

The corresponding Kraus operators have the following form:

$$\hat{E}_0(t) = e^{-t[\Gamma/2+im]} |\pi^0\rangle \langle \pi^0| + |0\rangle \langle 0|, \quad (19a)$$

$$\hat{E}_1(t) = \sqrt{1 - e^{-t\Gamma}} |0\rangle \langle \pi^0|, \quad (19b)$$

while the generators of the master equation are

$$\hat{H} = m |\pi^0\rangle \langle \pi^0|, \quad (20a)$$

$$\hat{L}_1 = \sqrt{\Gamma} |0\rangle \langle \pi^0|. \quad (20b)$$

Note that the evolution of π^0 is thus simply the amplitude damping quantum operation [17] with the probability of damping depending on time, namely $p = 1 - e^{-t\Gamma}$. If the initial state of the system is $\hat{\rho}(0) = |\pi^0\rangle \langle \pi^0|$, then, as expected, the probability of detecting π^0 at the time t is

$$p(\pi^0) = \text{tr}[\hat{\rho}(t) |\pi^0\rangle \langle \pi^0|] = e^{-t\Gamma}, \quad (21)$$

i.e., it is given by the Geiger–Nuttall law.

B. The time evolution of K^0 (B^0)

Now, we consider the case of a K^0 (B^0) meson. This particle needs special treatment because during the time evolution it transmutes into its antiparticle. Because both K^0 and B^0 mesons evolve according to the same scheme, hereafter we shall deal with K^0 , but the results are also valid for B^0 after appropriate changes of notation.

The Hilbert space of the kaon–vacuum system $\mathcal{H}_{K^0} \oplus \mathcal{H}_0$ is spanned by orthonormal vectors $|K^0\rangle$, $|\bar{K}^0\rangle$, and $|0\rangle$, which are the eigenstates of the strangeness operator \hat{S} :

$$\hat{S}|K^0\rangle = |K^0\rangle, \quad \hat{S}|\bar{K}^0\rangle = -|\bar{K}^0\rangle, \quad \hat{S}|0\rangle = 0. \quad (22)$$

These states (except of $|0\rangle$) are not eigenstates of the operator $\hat{C}\hat{P}$, where \hat{P} is the space reflection and \hat{C} is the charge conjugation. The $\hat{C}\hat{P}$ eigenstates $|K_1^0\rangle$ and $|K_2^0\rangle$,

$$\hat{C}\hat{P}|K_1^0\rangle = |K_1^0\rangle, \quad \hat{C}\hat{P}|K_2^0\rangle = -|K_2^0\rangle, \quad \hat{C}\hat{P}|0\rangle = |0\rangle, \quad (23)$$

are related to \hat{S} eigenstates by

$$|K_1^0\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle), \quad (24a)$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle). \quad (24b)$$

On the other hand, the time evolution operator is not diagonal in the basis (23) due to CP violation, but it is diagonal in the basis (see, e.g., [26])

$$|K_S^0\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (|K_1^0\rangle + \varepsilon|K_2^0\rangle), \quad (25a)$$

$$|K_L^0\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (\varepsilon|K_1^0\rangle + |K_2^0\rangle), \quad (25b)$$

with ε being the complex CP -violation parameter, $|\varepsilon| \approx 2.284 \times 10^{-3}$ [27]. The time evolution in this basis is assumed to follow the Weisskopf–Wigner phenomenological prescription,

$$|K_S^0(t)\rangle = e^{-t(im_S + \Gamma_S/2)} |K_S^0\rangle, \quad (26a)$$

$$|K_L^0(t)\rangle = e^{-t(im_L + \Gamma_L/2)} |K_L^0\rangle, \quad (26b)$$

where Γ_S and Γ_L are decay widths of K_S^0 and K_L^0 , respectively; m_S and m_L are some parameters—their physical meaning is provided by the formulas (39). We would like to point out that these masses cannot be the eigenvalues of a Hermitian Hamiltonian because of a CP violation. Indeed, the basis (25) is no longer orthonormal, since

$$\langle K_S^0 | K_L^0 \rangle = \frac{2\Re(\varepsilon)}{1+|\varepsilon|^2} \equiv \delta_L \approx 3.27 \times 10^{-3}, \quad (27)$$

and these states *cannot be the eigenstates of a Hermitian operator*.

The most convenient way of analyzing the evolution of the density operator is to decompose it as follows:

$$\begin{aligned} \hat{\rho}(t) = & \tilde{\rho}_{SS}(t) |K_S^0\rangle \langle K_S^0| + \tilde{\rho}_{SL}(t) |K_S^0\rangle \langle K_L^0| + \tilde{\rho}_{S0}(t) |K_S^0\rangle \langle 0| \\ & + \tilde{\rho}_{SL}^*(t) |K_L^0\rangle \langle K_S^0| + \tilde{\rho}_{LL}(t) |K_L^0\rangle \langle K_L^0| + \tilde{\rho}_{L0}(t) |K_L^0\rangle \langle 0| \\ & + \tilde{\rho}_{S0}^*(t) |0\rangle \langle K_S^0| + \tilde{\rho}_{L0}^*(t) |0\rangle \langle K_L^0| + \tilde{\rho}_{00}(t) |0\rangle \langle 0|. \end{aligned} \quad (28)$$

The superselection rule for a K^0 meson allows one to put $\tilde{\rho}_{S0}(t) = \tilde{\rho}_{L0}(t) = 0$ for physical states.

Because the basis (25) is nonorthogonal, the matrix $\tilde{\rho}(t)$ built from the coefficients of the decomposition (28) *is not formed from matrix elements of the density operator $\hat{\rho}(t)$ in this basis*, therefore one should be careful while operating on $\tilde{\rho}(t)$, especially $\text{tr}[\hat{\rho}(t)] = 1$ implies that

$$\text{tr}[\tilde{\rho}(t)] = 1 - 2\delta_L \Re[\tilde{\rho}_{SL}(t)]. \quad (29)$$

(see Appendix B).

Let us denote by $\rho(t)$ the matrix formed by matrix elements of $\hat{\rho}(t)$ in the orthonormal basis $\{|K_1^0\rangle, |K_2^0\rangle, |0\rangle\}$. From (25) it follows that these two matrix representations of $\hat{\rho}(t)$ are connected by

$$\rho(t) = V \tilde{\rho}(t) V^\dagger, \quad (30)$$

where

$$V = \begin{pmatrix} (1+|\varepsilon|^2)^{-1/2} & \varepsilon(1+|\varepsilon|^2)^{-1/2} & 0 \\ \varepsilon(1+|\varepsilon|^2)^{-1/2} & (1+|\varepsilon|^2)^{-1/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (31)$$

Now, we find the time evolution of the density operator in terms of the matrix $\tilde{\rho}$. Using (30) we can write

$$\rho(t) = \mathcal{S}_t \rho(0) = \mathcal{S}_t V \tilde{\rho}(0) V^\dagger = V \tilde{\mathcal{S}}_t \tilde{\rho}(0) V^\dagger, \quad (32)$$

where $\tilde{\rho}(t) = \tilde{\mathcal{S}}_t \tilde{\rho}(0)$ [cf. (30)].

The maps \mathcal{S}_t must be completely positive and must form a one-parameter semigroup. By virtue of (32) the maps \mathcal{S}_t are composed from $\tilde{\mathcal{S}}_t$ and the map (30). One can easily check using Choi's theorem that the map (30) is completely positive, so the maps \mathcal{S}_t are completely positive iff the maps $\tilde{\mathcal{S}}_t$ are also completely positive. It is much easier to find the conditions under which the latter maps are completely positive.

After checking the conditions for complete positivity (see Appendix A) and taking into account the superselection rule, we get the following time evolution of the matrix $\tilde{\rho}(t)$:

$$\tilde{\rho}(t) = \begin{pmatrix} e^{-t\Gamma_S} \tilde{\rho}_{SS}(0) & e^{-t(\Gamma+\lambda-i\Delta m)} \tilde{\rho}_{SL}(0) & 0 \\ e^{-t(\Gamma+\lambda+i\Delta m)} \tilde{\rho}_{SL}^*(0) & e^{-t\Gamma_L} \tilde{\rho}_{LL}(0) & 0 \\ 0 & 0 & \tilde{\rho}_{00}(t) \end{pmatrix}, \quad (33)$$

where $\Delta m = m_L - m_S$ (see later), $\Gamma = (\Gamma_S + \Gamma_L)/2$, and because of (29),

$$\begin{aligned} \tilde{\rho}_{00}(t) = & (1 - e^{-t\Gamma_S}) \tilde{\rho}_{SS}(0) + (1 - e^{-t\Gamma_L}) \tilde{\rho}_{LL}(0) \\ & + 2\delta_L \Re[(1 - e^{-t(\Gamma+\lambda-i\Delta m)}) \tilde{\rho}_{SL}(0)] + \tilde{\rho}_{00}(0). \end{aligned} \quad (33a)$$

Note that, contrary to the case of π^0 , the decoherence parameter λ is no longer irrelevant for evolution of physical states.

The condition that the evolution of the density operator should be completely positive requires that the inequality [see (A9)],

$$\delta_L^2 (1 - 2e^{-t(\Gamma+\lambda)} \cos(t\Delta m) + e^{-2t(\Gamma+\lambda)}) \leq (1 - e^{-t\Gamma_S})(1 - e^{-t\Gamma_L}), \quad (34)$$

must be valid for any $t \geq 0$. This inequality restricts the range of the parameters λ , Γ_S , Γ_L , Δm , and δ_L . For the physical values of Γ_S , Γ_L , Δm , and δ_L for K^0 mesons and corresponding parameters for B^0 mesons, inequality (34) implies an upper bound on λ (see Sec. III).

We can write the evolution of the density operator given by (28) and (33) in the form of the operator-sum representation (13) with the following Kraus operators (see Appendix B):

$$\begin{aligned} \hat{E}_0(t) = & \frac{1}{1 - \delta_L^2} \left[e^{-t[(\Gamma_S+\lambda)/2+im_S]} |K_S^0\rangle \langle K_S^0| \right. \\ & + e^{-t[(\Gamma_L+\lambda)/2+im_L]} |K_L^0\rangle \langle K_L^0| - \delta_L \left(e^{-t[(\Gamma_S+\lambda)/2+im_S]} |K_S^0\rangle \langle K_L^0| \right. \\ & \left. \left. + e^{-t[(\Gamma_L+\lambda)/2+im_L]} |K_L^0\rangle \langle K_S^0| \right) \right] + e^{-t\lambda/2} |0\rangle \langle 0|, \end{aligned} \quad (35a)$$

$$\hat{E}_1(t) = \frac{1}{1 - \delta_L^2} \sqrt{1 - e^{-t\Gamma_S} - \delta_L^2 \frac{|1 - e^{-t(\Gamma + \lambda - i\Delta m)}|^2}{1 - e^{-t\Gamma_L}}} (|0\rangle\langle K_S^0| - \delta_L |0\rangle\langle K_L^0|), \quad (35b)$$

$$\hat{E}_2(t) = \frac{1}{1 - \delta_L^2} \left[\left(\sqrt{1 - e^{-t\Gamma_L}} - \delta_L^2 \frac{1 - e^{-t(\Gamma + \lambda - i\Delta m)}}{\sqrt{1 - e^{-t\Gamma_L}}} \right) |0\rangle\langle K_L^0| - \delta_L \left(\sqrt{1 - e^{-t\Gamma_L}} - \frac{1 - e^{-t(\Gamma + \lambda - i\Delta m)}}{\sqrt{1 - e^{-t\Gamma_L}}} \right) |0\rangle\langle K_S^0| \right], \quad (35c)$$

$$\hat{E}_3(t) = \frac{e^{-t\Gamma_S/2} \sqrt{1 - e^{-t\lambda}}}{1 - \delta_L^2} (|K_S^0\rangle\langle K_S^0| - \delta_L |K_S^0\rangle\langle K_L^0|), \quad (35d)$$

$$\hat{E}_4(t) = \frac{e^{-t\Gamma_L/2} \sqrt{1 - e^{-t\lambda}}}{1 - \delta_L^2} (|K_L^0\rangle\langle K_L^0| - \delta_L |K_L^0\rangle\langle K_S^0|), \quad (35e)$$

$$\hat{E}_5(t) = \sqrt{1 - e^{-t\lambda}} |0\rangle\langle 0|. \quad (35f)$$

Note that (34) ensures the reality of the square root in (35b).

The density operator $\hat{\rho}(t)$ fulfills the master equation (15) with the following Lindblad operators:

$$\hat{L}_1 = \frac{\sqrt{\Gamma_S - \delta_L^2 |\Gamma + \lambda - i\Delta m|^2 / \Gamma_L}}{1 - \delta_L^2} (|0\rangle\langle K_S^0| - \delta_L |0\rangle\langle K_L^0|), \quad (36a)$$

$$\hat{L}_2 = \frac{\sqrt{\Gamma_L} - \delta_L^2 (\Gamma + \lambda - i\Delta m) / \sqrt{\Gamma_L}}{1 - \delta_L^2} |0\rangle\langle K_L^0| - \delta_L \frac{\sqrt{\Gamma_L} - (\Gamma + \lambda - i\Delta m) / \sqrt{\Gamma_L}}{1 - \delta_L^2} |0\rangle\langle K_S^0|, \quad (36b)$$

$$\hat{L}_3 = \frac{\sqrt{\lambda}}{1 - \delta_L^2} (|K_S^0\rangle\langle K_S^0| - \delta_L |K_S^0\rangle\langle K_L^0|), \quad (36c)$$

$$\hat{L}_4 = \frac{\sqrt{\lambda}}{1 - \delta_L^2} (|K_L^0\rangle\langle K_L^0| - \delta_L |K_L^0\rangle\langle K_S^0|), \quad (36d)$$

$$\hat{L}_5 = \sqrt{\lambda} |0\rangle\langle 0|, \quad (36e)$$

and with the Hamiltonian of the form (this is exactly the Hermitian part of the Weisskopf–Wigner Hamiltonian):

$$\hat{H} = \frac{1}{1 - \delta_L^2} \{ m_S |K_S^0\rangle\langle K_S^0| + m_L |K_L^0\rangle\langle K_L^0| - \delta_L [(m - i\Delta\Gamma/4) |K_S^0\rangle\langle K_L^0| + (m + i\Delta\Gamma/4) |K_L^0\rangle\langle K_S^0|] \}, \quad (37)$$

where $\Delta\Gamma = \Gamma_S - \Gamma_L$ and $m = (m_L + m_S)/2$ is the mean K^0 mass (measured experimentally; $m = 497.648 \text{ MeV}/c^2$ for K^0 , $m_{B^0} = 5279.4 \text{ MeV}/c^2$ for B^0 [27]):

$$m_{K^0} = \langle K^0 | \hat{H} | K^0 \rangle = m, \quad (38a)$$

$$m_{\bar{K}^0} = \langle \bar{K}^0 | \hat{H} | \bar{K}^0 \rangle = m. \quad (38b)$$

Note that $m_{K^0} = m_{\bar{K}^0}$, as it is required by *CPT* theorem. Δm is measured by an observation of K^0 flavor oscillation. This finally gives us the interpretation of m_S and m_L , which appeared in (33), as the expectation values of Hamiltonian in $|K_L^0\rangle$ and $|K_S^0\rangle$ states:

$$m_L = \langle K_L^0 | \hat{H} | K_L^0 \rangle = m + \Delta m/2, \quad (39a)$$

$$m_S = \langle K_S^0 | \hat{H} | K_S^0 \rangle = m - \Delta m/2. \quad (39b)$$

We would like to stress that the $|K_S^0\rangle$ and $|K_L^0\rangle$ are *not eigenstates* of the Hamiltonian (37).

From (33), it follows that the probabilities of detecting K^0 and \bar{K}^0 are given by

$$p_{K^0}(t) = \text{tr}[\hat{\rho}(t) |K^0\rangle\langle K^0|] = \frac{1 + \delta_L}{2} \{ e^{-t\Gamma_S} \tilde{\rho}_{SS}(0) + e^{-t\Gamma_L} \tilde{\rho}_{LL}(0) + 2\Re [e^{-t(\Gamma + \lambda - i\Delta m)} \tilde{\rho}_{SL}(0)] \}, \quad (40a)$$

$$p_{\bar{K}^0}(t) = \text{tr}[\hat{\rho}(t) |\bar{K}^0\rangle\langle \bar{K}^0|] = \frac{1 - \delta_L}{2} \{ e^{-t\Gamma_S} \tilde{\rho}_{SS}(0) + e^{-t\Gamma_L} \tilde{\rho}_{LL}(0) - 2\Re [e^{-t(\Gamma + \lambda - i\Delta m)} \tilde{\rho}_{SL}(0)] \}. \quad (40b)$$

If the initial state is $|K^0\rangle$, these probabilities are

$$p_{K^0}(t) = \frac{1}{4} [e^{-t\Gamma_S} + e^{-t\Gamma_L} + 2e^{-t(\Gamma + \lambda)} \cos(t\Delta m)], \quad (41a)$$

$$p_{\bar{K}^0}(t) = \frac{1}{4} \frac{1 - \delta_L}{1 + \delta_L} [e^{-t\Gamma_S} + e^{-t\Gamma_L} - 2e^{-t(\Gamma + \lambda)} \cos(t\Delta m)]. \quad (41b)$$

The strangeness operator for K^0 is $\hat{S} = |K^0\rangle\langle K^0| - |\bar{K}^0\rangle\langle \bar{K}^0|$, and its average is

$$\langle \hat{S} \rangle = \text{tr}[\hat{\rho}(t) \hat{S}] = \delta_L [e^{-t\Gamma_S} \tilde{\rho}_{SS}(0) + e^{-t\Gamma_L} \tilde{\rho}_{LL}(0)] + 2\Re [e^{-t(\Gamma + \lambda - i\Delta m)} \tilde{\rho}_{SL}(0)]. \quad (42)$$

Regardless of the initial state $\lim_{t \rightarrow \infty} \langle \hat{S} \rangle = 0$, as expected, because in the limit $t \rightarrow \infty$ we have vacuum only. If the initial state is $|K^0\rangle$, we get

$$\langle \hat{S} \rangle = \frac{1}{1 + \delta_L} \left(e^{-t(\Gamma + \lambda)} \cos(t\Delta m) + \frac{\delta_L}{2} (e^{-t\Gamma_S} + e^{-t\Gamma_L}) \right). \quad (43)$$

Finally, we would like to point out that we are dealing with the Hermitian Hamiltonian and unit-trace density operator. This assures us that there is no ambiguity in calculating either conditional or joint probabilities for the results of measurements performed on the system. Moreover, we can unambiguously determine the states after the projective measurement using the standard quantum mechanical procedures (i.e., the von Neumann postulate of the state reduction).

III. CP VIOLATION AND DECOHERENCE

Now, let us analyze the inequality (34) in more detail. Taking into account that $\lambda \geq 0$, we can treat (34) as a quadratic

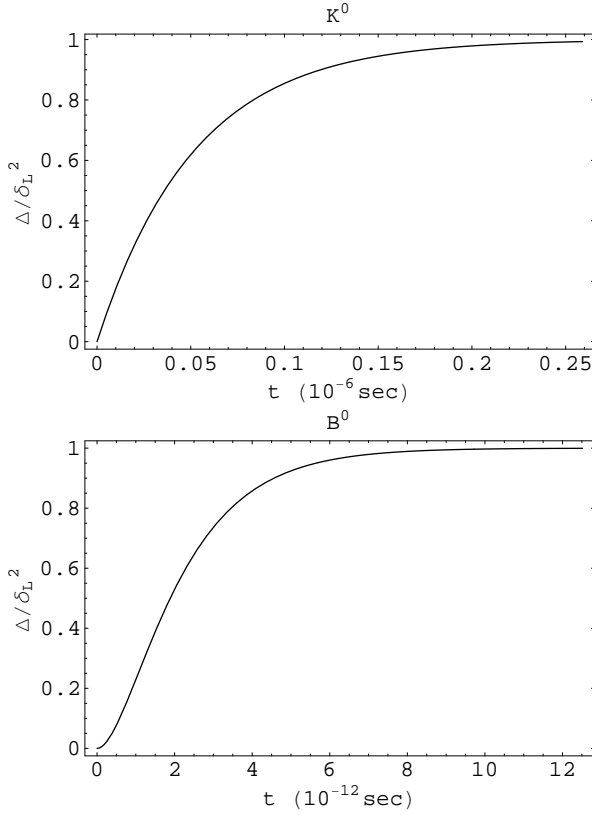


FIG. 1: The discriminant of inequality (34) as a function of time for K^0 and B^0 ; in both cases $\Delta \geq 0$, implying the validity of (44).

inequality in $e^{-t(\Gamma+\lambda)}$. This inequality has real solutions provided that its discriminant Δ fulfills

$$\Delta/\delta_L^2 = [(1 - e^{-t\Gamma_S})(1 - e^{-t\Gamma_L}) - \delta_L^2 \sin^2(t\Delta m)] \geq 0 \quad (44)$$

for any $t \geq 0$, where, according to [27]: $\Delta m = 0.5292 \times 10^{10} \text{ s}^{-1}$, $\tau_S \equiv 1/\Gamma_S = 0.8953 \times 10^{-10} \text{ s}$, $\tau_L \equiv 1/\Gamma_L = 5.18 \times 10^{-8} \text{ s}$. $\Delta m_{B^0} = 0.502 \times 10^{12} \text{ s}^{-1}$ ($\Delta m_{B^0} \approx m_{B_H^0} - m_{B^0}$), $\tau \equiv 1/\Gamma = 1.536 \times 10^{-12} \text{ s}$, and $\Re(\epsilon_{B^0})/(1 + |\epsilon_{B^0}|^2) = 0.5 \times 10^{-3}$. Fortunately, the inequality (44) holds for any $t \geq 0$ for both K^0 and B^0 mesons because the first term rapidly grows from 0 to 1 while the second one oscillates between 0 and $\delta_L^2 \sim 10^{-6}$ (see Fig. 1). Indeed, the series expansion of (44) is

$$\Delta/\delta_L^2 \simeq [\Gamma_S \Gamma_L - \delta_L^2 (\Delta m)^2] t^2 + O(t^3). \quad (45)$$

Therefore the condition

$$\delta_L \leq \frac{\sqrt{\Gamma_S \Gamma_L}}{\Delta m} \quad (46)$$

is necessary for the existence of the solutions of inequality (34). This condition is satisfied for both K^0 and B^0 mesons.

Now, let us analyze the restrictions imposed by the inequality (34) on the range of the decoherence parameter λ . From (34) we have, for any $t \geq 0$,

$$\cos(t\Delta m) - \frac{\sqrt{\Delta}}{\delta_L^2} \leq e^{-t(\Gamma+\lambda)} \leq \cos(t\Delta m) + \frac{\sqrt{\Delta}}{\delta_L^2}. \quad (47)$$

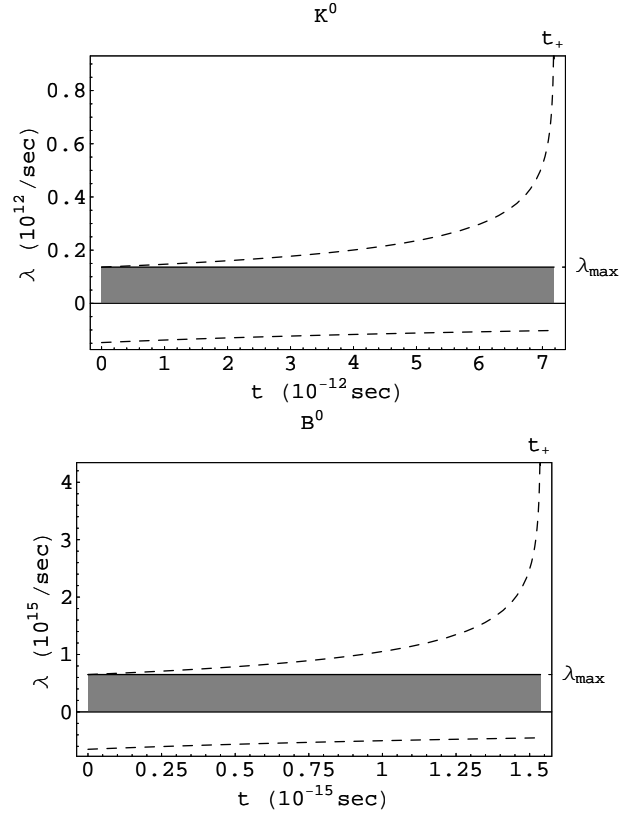


FIG. 2: The allowed region for the decoherence parameter (gray) for K^0 and B^0 mesons: dashed curves are the upper and lower bound from (48); the lower bound is always negative. The values of t_+ and λ_{\max} for K^0 and B^0 are given in the text.

The left inequality gives a restriction only when its left-hand side is positive, which holds for K^0 and B^0 only for $t \leq t_+$, where $t_+ \approx 7.18517 \times 10^{-12} \text{ s}$ for K^0 and $t_+ \approx 1.53677 \times 10^{-15} \text{ s}$ for B^0 . Therefore

$$\lambda \leq -\frac{1}{t} \ln[\cos(t\Delta m) - \sqrt{\Delta}] - \Gamma, \quad (48a)$$

$$\lambda \geq -\frac{1}{t} \ln[\cos(t\Delta m) + \sqrt{\Delta}] - \Gamma, \quad (48b)$$

where the upper inequality must hold for $0 \leq t \leq t_+$ and the lower one for any $t \geq 0$; moreover, as mentioned earlier, $\lambda \geq 0$. These bounds are presented in Fig. 2 for K^0 and B^0 . Thus, for K^0 and B^0 ,

$$0 \leq \lambda \leq \lambda_{\max} = \inf_{0 \leq t \leq t_+} \left(-\frac{1}{t} \ln[\cos(t\Delta m) - \sqrt{\Delta}] - \Gamma \right) \simeq \frac{1}{\delta_L} \sqrt{\Gamma_S \Gamma_L - \delta_L^2 (\Delta m)^2} - \Gamma, \quad (49)$$

where the last equality comes from the first-order expansion. For K^0 we get $\lambda_{\max} = 1.3629 \times 10^{11} \text{ s}^{-1}$ and for B^0 we get $\lambda_{\max} = 6.5039 \times 10^{14} \text{ s}^{-1}$. The experimental value for the decoherence parameter in an entangled $K^0 \bar{K}^0$ system is $\lambda = (1.84_{-2.17}^{+2.50}) \times 10^{-12} \text{ MeV} = 2.80_{-3.30}^{+3.80} \times 10^9 \text{ s}^{-1}$ [28] and for a $B^0 \bar{B}^0$ system is $\lambda = (-47 \pm 76) \times 10^{-12} \text{ MeV} =$

$(-0.71 \pm 1.15) \times 10^{11} \text{ s}^{-1}$ [29] and they fit in the allowed range.

IV. CONCLUSIONS

In this paper, we have found the probability-preserving description of the decaying particle within the framework of quantum mechanics of open systems, taking into account the superselection rule prohibiting the superposition of the particle and vacuum. It has been shown that some limitations of the Weisskopf–Wigner approach can be removed if we assume that the particle can be found in one of its possible states as well as in the state of *the absence of the particle*, i.e., in the *vacuum* state. In our approach the evolution of the system is given by a family of completely positive trace-preserving maps forming a one-parameter dynamical semigroup; thus the *Hamiltonian is Hermitian* and therefore the *reduced density operator has a unit trace*. It should be noted that we have used the dynamical semigroup approach for the description of the *entire unstable particle*, not only for the description of its decoherence, as in [3, 13, 14, 15, 16]. The advantage of the introduced approach is that there is no ambiguity in calculating either conditional or joint probabilities for the results of measurements performed on the system. Furthermore, we can unambiguously determine the states after the measurement using the standard quantum mechanical procedures (i.e., the von Neumann postulate on the state reduction). We have also shown that there are restrictions on the possible strength of decoherence that arise as a remarkable side effect of completely positive evolution of the particle. Moreover, we have found the operator sum representation (Kraus representation)

for the general evolution of such systems, which allows one to write the evolution for systems with two or more particles. This is extremely useful if we study quantum correlations between unstable particles. Moreover, we have shown that the decay of the particle can be regarded as a Markov process by finding explicitly the Lindblad form of the master equation for such a system.

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APPENDIX A: TIME DEPENDENCE OF DENSITY MATRICES FOR K^0

In this appendix we show that the evolution given by (33) is the most general completely positive trace-preserving linear map that possesses the semigroup property and leads to the Weisskopf–Wigner evolution (26). This implies that we have well-defined $\tilde{\rho}_{SS}(t) = \tilde{\rho}_{SS}(0)e^{-t\Gamma_S}$ and $\tilde{\rho}_{LL}(t) = \tilde{\rho}_{LL}(0)e^{-t\Gamma_L}$. Because kaons carry some quantum numbers like strangeness, we impose the superselection rule from the very beginning.

The most general form of the evolution can be written in the form

$$\tilde{\mathcal{S}}_t \tilde{\rho}(0) = \begin{pmatrix} e^{-t\Gamma_S} \tilde{\rho}_{SS}(0) & \sum_{i,j=S,L,0} A_{ij}(t) \tilde{\rho}_{ij}(0) & \sum_{i,j=S,L,0} B_{ij}(t) \tilde{\rho}_{ij}(0) \\ \sum_{i,j=S,L,0} A_{ij}^*(t) \tilde{\rho}_{ij}^*(0) & e^{-t\Gamma_L} \tilde{\rho}_{LL}(0) & \sum_{i,j=S,L,0} C_{ij}(t) \tilde{\rho}_{ij}(0) \\ \sum_{i,j=S,L,0} B_{ij}^*(t) \tilde{\rho}_{ij}^*(0) & \sum_{i,j=S,L,0} C_{ij}^*(t) \tilde{\rho}_{ij}^*(0) & \sum_{i,j=S,L,0} D_{ij}(t) \tilde{\rho}_{ij}(0) \end{pmatrix}. \quad (\text{A1})$$

The superselection rule causes that the only nonvanishing B 's and C 's are $B_{i0}(t)$, $B_{0i}(t)$, $C_{i0}(t)$, and $C_{0i}(t)$; moreover, since $\tilde{\rho}_{00}(t)$ must be real, $D_{LS}(t) = D_{SL}^*(t)$, $D_{0S}(t) = D_{S0}^*(t)$, $D_{0L}(t) = D_{L0}^*(t)$, and the other D 's are real functions. The initial conditions are $A_{SL}(0) = B_{S0}(0) = C_{L0}(0) = 1$ and the other functions vanish at $t = 0$; moreover, $D_{00}(t) \equiv 1$ because

the vacuum must be a fixed point of this dynamics, i.e.,

$$\rho(0) = |0\rangle\langle 0| \Rightarrow \forall t \geq 0: \rho(t) = |0\rangle\langle 0|. \quad (\text{A2})$$

The corresponding Choi's matrix is

$$\text{Choi } \tilde{\mathcal{S}}_t = \begin{pmatrix} e^{-t\Gamma_S} & A_{SS}(t) & 0 & 0 & A_{SL}(t) & 0 & 0 & A_{S0}(t) & B_{S0}(t) \\ A_{SS}^*(t) & 0 & 0 & A_{LS}^*(t) & 0 & 0 & A_{0S}^*(t) & 0 & C_{S0}(t) \\ 0 & 0 & D_{SS}(t) & 0 & 0 & D_{SL}(t) & B_{0S}^*(t) & C_{0S}^*(t) & D_{S0}(t) \\ 0 & A_{LS}(t) & 0 & 0 & A_{LL}(t) & 0 & 0 & A_{L0}(t) & B_{L0}(t) \\ A_{SL}^*(t) & 0 & 0 & A_{LL}^*(t) & e^{-t\Gamma_L} & 0 & A_{0L}^*(t) & 0 & C_{L0}(t) \\ 0 & 0 & D_{SL}^*(t) & 0 & 0 & D_{LL}(t) & B_{0L}^*(t) & C_{0L}^*(t) & D_{L0}(t) \\ 0 & A_{0S}(t) & B_{0S}(t) & 0 & A_{0S}(t) & B_{0L}(t) & 0 & A_{00}(t) & B_{00}(t) \\ A_{S0}^*(t) & 0 & C_{0S}(t) & A_{L0}^*(t) & 0 & C_{0L}(t) & A_{00}^*(t) & 0 & C_{00}(t) \\ B_{S0}^*(t) & C_{S0}^*(t) & D_{S0}^*(t) & B_{L0}^*(t) & C_{L0}^*(t) & D_{L0}^*(t) & B_{00}^*(t) & C_{00}^*(t) & 1 \end{pmatrix} \quad (\text{A3})$$

The positivity of this matrix requires that from the functions A 's, B 's, and C 's the only nonvanishing functions are $A_{SL}(t)$, $B_{S0}(t)$, and $C_{L0}(t)$. Moreover, the condition on the trace of $\hat{\rho}(t)$ implies that $\text{tr}[\hat{\rho}(t)] = \text{tr}[V\tilde{\rho}(t)V^\dagger] = 1$, so we have

$$\begin{aligned} & [e^{-t\Gamma_S} + D_{SS}(t)]\tilde{\rho}_{SS}(0) + [e^{-t\Gamma_L} + D_{LL}(t)]\tilde{\rho}_{LL}(0) \\ & + 2\Re\{[\delta_L A_{SL}(t) + D_{SL}(t)]\tilde{\rho}_{SL}(0)\} \\ & + 2\Re[D_{S0}(t)\tilde{\rho}_{S0}(0)] + 2\Re[D_{L0}(t)\tilde{\rho}_{L0}(0)] \\ & = \tilde{\rho}_{SS}(0) + \tilde{\rho}_{LL}(0) + 2\delta_L\Re[\tilde{\rho}_{SL}(0)]. \end{aligned} \quad (\text{A4})$$

Because this equation must be valid for any $\hat{\rho}(0)$, this implies that $D_{i0}(t) = 0$ for $i \neq 0$, and

$$D_{00}(t) \equiv 1, \quad (\text{A5a})$$

$$D_{SS}(t) = 1 - e^{-t\Gamma_S}, \quad (\text{A5b})$$

$$D_{LL}(t) = 1 - e^{-t\Gamma_L}, \quad (\text{A5c})$$

$$D_{SL}(t) = \delta_L[1 - A_{SL}(t)]. \quad (\text{A5d})$$

Therefore, the matrix (A3) is positive, provided that

$$|A_{SL}(t)|^2 \leq e^{-t(\Gamma_S + \Gamma_L)}, \quad (\text{A6a})$$

$$|B_{S0}(t)|^2 \leq e^{-t\Gamma_L}, \quad (\text{A6b})$$

$$|C_{L0}(t)|^2 \leq e^{-t\Gamma_S}, \quad (\text{A6c})$$

$$\begin{aligned} & |B_{S0}(t)|^2 e^{-t\Gamma_L} + |C_{L0}(t)|^2 e^{-t\Gamma_S} + |A_{SL}(t)|^2 \\ & - 2\Re[A_{SL}(t)B_{S0}^*(t)C_{L0}(t)] \leq e^{-t(\Gamma_S + \Gamma_L)}, \end{aligned} \quad (\text{A6d})$$

and

$$|D_{SL}(t)|^2 \leq D_{SS}(t)D_{LL}(t). \quad (\text{A6e})$$

Taking into account the composition law for the \mathcal{S}_t super-operator and (A6a)–(A6c), we get

$$A_{SL}(t) = e^{-t[(\Gamma_S + \Gamma_L)/2 + a - i\mu_{SL}]}, \quad (\text{A7a})$$

$$B_{S0}(t) = e^{-t[\Gamma_S/2 + b + i\mu_S]}, \quad (\text{A7b})$$

$$C_{L0}(t) = e^{-t[\Gamma_L/2 + c + i\mu_L]}, \quad (\text{A7c})$$

where $a, b, c, \mu_{SL}, \mu_S, \mu_L \geq 0$. Now, one can observe that the concrete time dependence of the functions $B_{S0}(t)$ and $C_{L0}(t)$ is irrelevant, since the superselection rule requires that $\tilde{\rho}_{S0}(0) =$

$\tilde{\rho}_{L0}(0) = 0$ and, therefore, $\tilde{\rho}_{S0}(t)$ and $\tilde{\rho}_{L0}(t)$ still remain vanishing (as required), so, without loss of any generality, we can choose $a = b = c = \lambda$ and $\mu_S = m_S$, $\mu_L = m_L$, and then $\mu_{SL} = \Delta m$. Thus, finally, we can put

$$A_{SL}(t) = e^{-t[(\Gamma_S + \Gamma_L)/2 + \lambda - i\Delta m]}, \quad (\text{A8a})$$

$$B_{S0}(t) = e^{-t[\Gamma_S/2 + \lambda + im_S]}, \quad (\text{A8b})$$

$$C_{L0}(t) = e^{-t[\Gamma_L/2 + \lambda + im_L]}, \quad (\text{A8c})$$

and, consequently,

$$D_{SL}(t) = \delta_L \left(1 - e^{-t[(\Gamma_S + \Gamma_L)/2 + \lambda - i\Delta m]} \right). \quad (\text{A8d})$$

Now, taking the solutions (A5a)–(A5c) and (A8), one can easily see that among conditions (A6) for positivity of the matrix $\text{Choi } \tilde{\mathcal{S}}_t$ the only one that is not identically fulfilled is (A6e); it can be written in the form

$$\delta_L^2 \left| 1 - e^{-t[(\Gamma_S + \Gamma_L)/2 + \lambda - i\Delta m]} \right|^2 \leq (1 - e^{-t\Gamma_S})(1 - e^{-t\Gamma_L}), \quad (\text{A9})$$

for any $t \geq 0$.

APPENDIX B: OPERATOR SUM REPRESENTATION FOR EVOLUTION OF K^0

Let us denote

$$g \equiv V^\dagger V = \begin{pmatrix} 1 & \delta_L & 0 \\ \delta_L & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (\text{B1})$$

We have

$$\text{tr}[\hat{\rho}(t)] = \text{tr}[\tilde{\rho}(t)g], \quad (\text{B2})$$

and therefore we can easily find Eq. (29).

Now, let us find the operator sum representation for the evolution given by the map (33). Let us define the set of matrices $\check{E}_i(t)$, such that

$$\tilde{\rho}(t) = \sum_i \check{E}_i(t)\tilde{\rho}(0)\check{E}_i^\dagger(t). \quad (\text{B3})$$

After a little algebra, we find that the matrices $\check{E}_i(t)$ are

$$\check{E}_0(t) = \begin{pmatrix} e^{-t[(\Gamma_S+\lambda)/2+im_S]} & 0 & 0 \\ 0 & e^{-t[(\Gamma_L+\lambda)/2+im_L]} & 0 \\ 0 & 0 & e^{-t\lambda/2} \end{pmatrix}, \quad (\text{B4a})$$

$$\check{E}_1(t) = \sqrt{1 - e^{-t\Gamma_S} - \delta_L^2 \frac{|1 - e^{-t(\Gamma+\lambda-i\Delta m)}|^2}{1 - e^{-t\Gamma_L}}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (\text{B4b})$$

$$\check{E}_2(t) = \sqrt{1 - e^{-t\Gamma_L}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \delta_L \frac{1 - e^{-t(\Gamma+\lambda-i\Delta m)}}{1 - e^{-t\Gamma_L}} & 1 & 0 \end{pmatrix}, \quad (\text{B4c})$$

$$\check{E}_3(t) = e^{-t\Gamma_S/2} \sqrt{1 - e^{-t\lambda}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (\text{B4d})$$

$$\check{E}_4(t) = e^{-t\Gamma_L/2} \sqrt{1 - e^{-t\lambda}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (\text{B4e})$$

$$\check{E}_5(t) = \sqrt{1 - e^{-t\lambda}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (\text{B4f})$$

These matrices can be helpful in finding the Kraus operators $\hat{E}_i(t)$. Indeed,

$$\sum_i V \check{E}_i(t) \tilde{\rho}(0) \check{E}_i^\dagger(t) V^\dagger = \rho(t) = \sum_i E_i(t) \rho(0) E_i^\dagger(t), \quad (\text{B5})$$

where matrices on the right-hand side are written in the orthonormal basis (23). This gives

$$E_i(t) = V \check{E}_i(t) V^{-1}. \quad (\text{B6})$$

More interesting than finding explicitly the matrices $E_i(t)$'s is finding the decomposition of $\hat{E}_i(t)$ into the sum

$$\begin{aligned} \hat{E}_i(t) = & \check{E}_i(t)_{SS} |K_S^0\rangle \langle K_S^0| + \check{E}_i(t)_{SL} |K_S^0\rangle \langle K_L^0| + \check{E}_i(t)_{S0} |K_S^0\rangle \langle 0| \\ & + \check{E}_i(t)_{LS} |K_L^0\rangle \langle K_S^0| + \check{E}_i(t)_{LL} |K_L^0\rangle \langle K_L^0| + \check{E}_i(t)_{L0} |K_L^0\rangle \langle 0| \\ & + \check{E}_i(t)_{0S} |0\rangle \langle K_S^0| + \check{E}_i(t)_{0L} |0\rangle \langle K_L^0| + \check{E}_i(t)_{00} |0\rangle \langle 0|. \end{aligned} \quad (\text{B7})$$

Using Eqs. (25) and (B6), we get finally that

$$\hat{E}_i(t) = V^{-1} E_i(t) V^{-1\dagger} = \check{E}_i(t) g^{-1}. \quad (\text{B8})$$

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